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Extended Common Load Model: A tool for dependent failure modelling in highly redundant structures

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SSM perspective

Background

The treatment of dependent failures is one of the most controversial subjects in reliability and risk analyses. The difficulties are specially underlined in the case of highly redundant systems, when the number of redundant components or trains exceeds four. The Common Load Model (CLM), originally defined in the 70'ies, differs from other CCF models, as it relies on a specific physical analogy. The model was extended in late 80'ies with specific aims to model highly redundant systems. Practical use of the model requires a computer tool, but in highly redundant systems the benefits give an evident payback.

Objectives

The extended method ECLM details have not been publicly available before. However, it has been developed with certain funding via the Nordic PSA group (NPSAG) including SSM. It has therefore been an interest to have the method properly documented and published. The objective with this report is to describe the mathematical details in the definition of Extended CLM (ECLM), which is developed to better suit for modelling of highly redundant systems. ECLM has four parameters, while the initial CLM was a two-parameter model. This report focuses on parameter definition and interface between a dedicated CCF analysis and PSA models. The estimation of model parameters is described for Maximum Likelihood and Bayesian approaches. Also a comparison is presented with respect to other common CCF models. Special extensions are described for time-dependent modelling and asymmetric CCF groups.

Results

The result is that the method is documented and available for the stakeholders, including SSM. The report is also published as an NPSAG report.

Need for further research

There is no continuation planned for this since it is the end result of previous work.

Project information

Contact person SSM: Per Hellström
Reference: NPSAG Project no. 34-002

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Summary

This report describes an extension of Common Load Model, specially developed for the treatment of failure probabilities and dependencies in highly redundant systems. The model is based on expressing failure condition by stress-resistance analogy; at the demand, the components are loaded by a common stress, and their failure is described by component resistances. A multiple failure occurs, when the load exceeds several component resistances. In Extended Common Load Model the load constitutes of base and extreme load parts, modelling failures at low and high orders, respectively. Four parameters are defined; for each part a probability parameter describing likelihood level and a correlation coefficient describing failure dependence. Good or reasonable fit has been obtained with dependence profiles in failure statistics even for large component groups. The estimation of model parameters can be based on standard procedures of maximum likelihood method or Bayesian method.

The model is defined in terms of subgroup failure probabilities and is subgroup invariant. This implies that the model applies with same parameter values to any subgroup of a homogeneous group, so called Common Cause Failure group. Subgroup invariance facilitated application of the model is varying demand conditions where different parts of the group are challenged. Furthermore, model parameters applicable to two separate groups of different size are directly comparable, if mutually homogeneous or under assumption of that, which is helpful also in data acquisition.

ECLM is basically defined as pure demand failure probability model. It can be extended to other types of failure on demand like instantaneous unavailability of standby components and failure over mission time. Further extensions have been made during the course of applications to model access criticality and failure correlation of adjacent control rods, and asymmetric groups like in case of design diversification, e.g. redundant components of old and new, and partially same design.

The main limitation is generally sparse statistical data about Common Cause Failures, a shared problem of parametric models for Common Cause Failures. The problem is pronounced with the model extensions where additional features should be verified by operational experience; much is based on engineering judgment for the time being.

Applications of ECLM concentrate in the Nordic BWR plants, for safety/relief valve systems, and control rods and drives. These applications have been supported by the collection of Common Cause Failure data for analysed systems. Applications to control rods and drives have been made also in French PWR plants. Other applications include highly redundant configurations of isolation valves and check valves, and main steam relief valves of a Russian RBMK plant.

Acknowledgements

The comments and support by the PSA team of Teollisuuden Voima Oy are acknowledged in the course of the initial development of the presented method and procedures for practical applications. The presentation of methodology has benefited from the comments by Aura Voicu, EdF in the connection to local applications. The valuable review comments by Sven Erick Alm, Uppsala University, and Christine Bell, AREVA NP GmbH, are also taken into account, especially aiding to improve the description of parameter definition. Jan-Erik Holmberg, Risk Pilot AB has greatly helped by advices and comments in the finalizing stage of this document.

Notice

Extended Common Load Model is implemented in HiDep Toolbox, developed by Avaplan Oy, containing various tools for the quantification of Common Cause Failures in different types of highly redundant systems. HiDep rights are now transferred to the following members of Nordic PSA Group: Forsmark Kraftgrupp AB, OKG AB, Ringhals AB and Teollisuuden Voima Oy which have also supported the elaboration of this document. Risk Pilot AB will maintain HiDep software and provide future user support. Generic HiDep Toolbox version is available to professional users under open software terms, contact hidep@riskpilot.se.

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1 Introduction

1.1 Characteristics of the CCF analysis for highly redundant systems

The treatment of dependent failures is one of the most controversial subjects in the reliability and risk analyses [CCF_PG, NKA/RAS-470]. The difficulties are specially underlined in the case of highly redundant systems, when the number of redundant components or trains exceeds four. The Common Load Model (CLM), originally defined in [CLM_77], differs from the other CCF models, as it relies on a specific physical analogy. This proves to be advantageous in several respects as will be explained in the continuation, and when comparing with the other approaches. The model was extended in late 80'ies with specific aims to model highly redundant systems. Practical use of the model requires a computer tool, but in highly redundant systems the benefits give an evident payback.

The motivation to this development arose from the practical needs to realistically assess the failure probability of the overpressure protection and pressure relief function in the PSA study at Teollisuuden Voima Oy (TVO). The TVO/BWR units (Olkiluoto 1 and 2) had twelve safety/relief valves (SRVs) at that time before modernization. The SRVs are imposed on demand in different ways depending on the transient case and event scenario. The success criteria range from 4/8 to 1..9/12. This early application has been described in [SRE88HiD]. TVO/PRA included also early applications to other highly redundant systems, including control rod and drive system.

1.2 Scope of this report

This report describes the mathematical details in the definition of Extended CLM (ECLM), which is developed to better suit for modelling of highly redundant systems. ECLM has four parameters, while the initial CLM was a two-parameter model. This report focuses on parameter definition and interface between a dedicated CCF analysis and PSA models. The estimation of model parameters is described for Maximum Likelihood and Bayesian approaches. Also a comparison is presented with respect to other common CCF models. Special extensions are described for time-dependent modelling and asymmetric CCF groups.

1.3 Applications

Since the early applications to TVO/PRA the following works can be highlighted:

- Analysis of SRV experience was carried out incorporating Swedish BWR units, and a reference application prepared for Forsmark 1/2, emphasis given on the integration with PSA models [SKI TR-91:6].

- CCF analysis of the hydraulic scram and control rod systems, including the analysis of operating experiences of the Nordic BWRs and reference application to Barsebäck 1 and 2 [SKI-R-96:77]
- Development of a time-dependent extension for risk monitor or follow-up purpose and test arrangement analysis of highly redundant systems [TVO_SRVX]
- Quantification of asymmetric system configurations for SRVs (for the modernized TVO plant and a new plant concept) and for containment isolation valves

Similar new applications have been made more recently for other plants, including following cases:

- Control rods and drives in French PWR plants [ICDE-EdF-2001]
- Highly redundant configurations of isolation valves and check valves, and main steam relief valves of Leningrad plant (RBMK).

Earlier applications have been upgraded, especially for control rods and drives [SKI Report 2006:05].

The practical applications have implied gradual development of the calculation tools. The treatment of critical failure combinations of control rods and drives has been enhanced for increased realism. Basic definition of ECLM has stayed as initial during the course of time and different kinds of applications.

2 Basic Concepts

To begin from, it is important to clearly specify, what is the measure of failure probability which is then primarily used to express dependencies among a group of components called as CCF group – or in the current terminology Common Cause Component Group (CCCG). The treatment of the subject is focused here to the quantification of time-independent failure probabilities or so called demand failure probabilities of standby components. A time-dependent extension of ECLM is discussed in Section 7.2.

2.1 CCF group and subgroup concept

Identical components of a CCF group, normally in standby, are imposed in the model, to an operation demand. The event of basic interest is the failure of specific 'm' components in the demand. The probability of such an event is denoted by

$$Psg(m) = P\{\text{Specific m components fail} | \text{Demand on CCF group n}\} \quad (2.1)$$

Particular variable notation "Psg" for Probability of SubGroup failure is used here, in order to make a clear distinction with respect to often confused probability notations.

The CCF group will be assumed internally homogeneous, hence Psg(m) is same for any choice of 'm' components, which constitute a subgroup of the considered CCF group. It is also assumed, that if the demand is placed only on a part of components, the failure probability of any challenged 'm' components will be same in the challenged subgroup, e.g. no extra stresses are imposed if fewer components participate in the system response. Such influences should be modelled separately. Especially, the total component failure probability Psg(1) is equal among the components, and remains same in different demand conditions. This aspect should not be confused with the treatment of event combinations which differ depending on the applicable failure criterion for the demand condition (to be discussed in Chapter 4).

An example of a highly redundant CCF group of ten electromagnetic pilot valves (EPVs) is presented in Figure 2-1. In a depressurization demand, a subgroup of eight EPVs are actuated to open the corresponding SRV main valves (which form a different CCF group). Four lines are sufficient to achieve the required fast pressure reduction. Consequently, if a subgroup of five or more EPVs fail within the challenged subgroup of eight EPVs, depressurization fails. An additional aspect influencing subgroup under focus is a situation where DC power supply to EPVs is degraded. For example, if one of the DC buses is down for maintenance (8 hour's Allowed Outage Time in power state applies at the Olkiluoto 1 and 2 plant), the depressurization function needs to be accomplished by a subgroup of six EPVs. The failure criterion changes from nominal 5 out of 8 into 3 out of 6.

An example of the critical failure combination when DC bus A is inoperable, and of the corresponding failure probability is:

$$Psg(3) = P\{X_{EPV181} * X_{EPV182} * X_{EPV184}\} \quad (2.2)$$

where $X_{EPV\#}$ denotes the failure of a specific EPV, compare to Figure 2-1. It should be emphasized, that the probability Psg(3) does not take into account the status of other 7 EPVs: they may operate or fail at demand, or may not be challenged at all if power or actuation signal is missing. Chapter 4 will explain, how the total failure probability of system function is derived from the basic probability entities Psg(m), which are preferred in the ECLM definition.

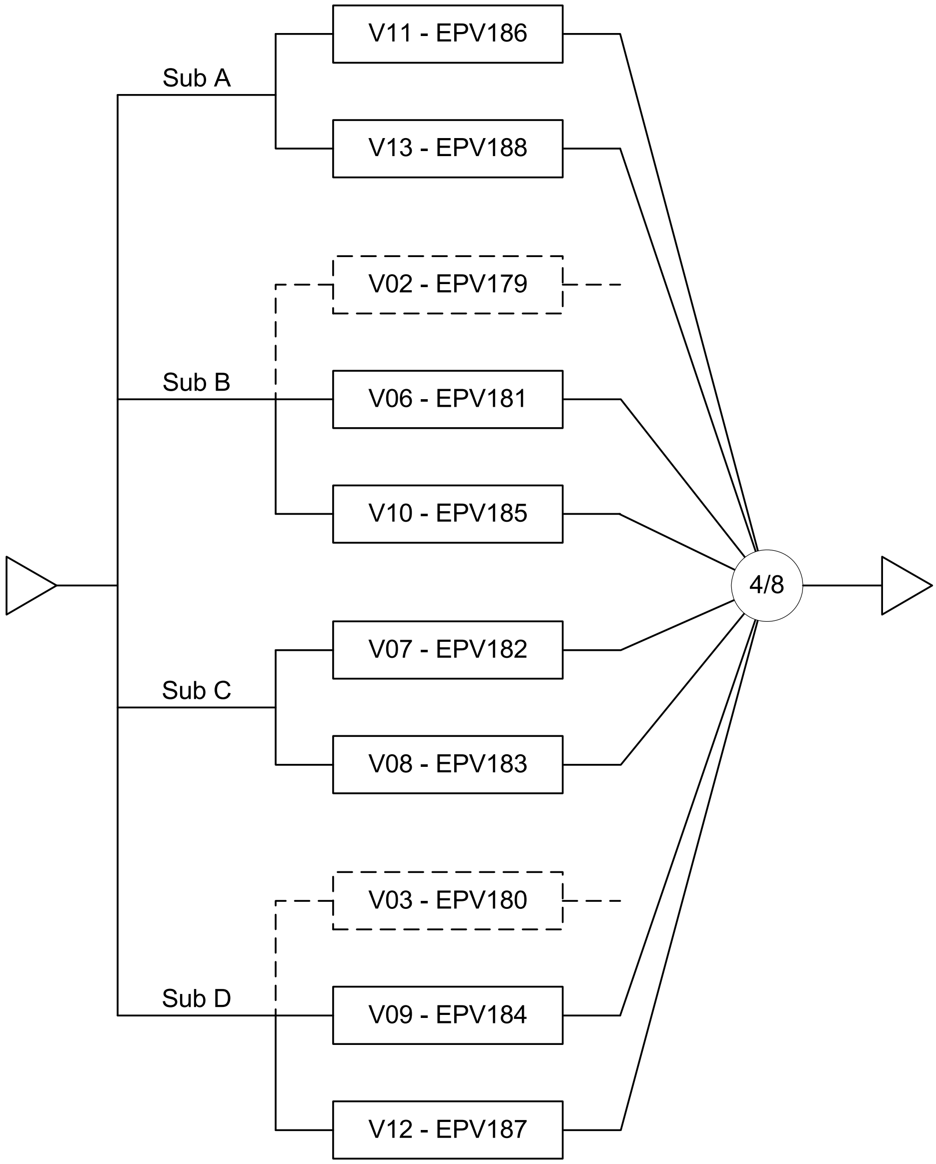


Figure 2-1 Illustration of a subgroup of size 8 in a CCF group comprising n=10 identical components for Olkiluoto 1 and 2 depressurization function (configuration before the plant modernization in 90'ies). In the depressurization demand 8 SRV lines are challenged: those SRV lines are presented by solid boxes, while the other two (which participate only in overpressure protection) are presented by dashed boxes. V02..13 denote main valves and EPV179..188 denote electromagnetic pilots. The success criterion is 4 out of 8 in this example, i.e. failure events of multiplicity 5 are minimal cut sets. The SRVs are equipped also with diverse impulse pilot valves that are spring-operated and provide backup actuation in overpressure protection. The impulse pilot valves are not part of remotely actuated depressurization function and are hence not presented in this simplified diagram.

Similarly, as for BWR reactor relief system, the success criteria vary for many safety systems depending on the initiating event and successful operation or failure of other systems. These different demand and success criteria cases need to be handled consistently. For this reason, it is beneficial, that the CCF quantification model applies to subgroups within the system with the same model parameters. This property is called subgroup invariance. It will be discussed in more detail in Chapter 4, along with its practical implications. The above defined Psg(m) entities are an example of subgroup invariant variables under the assumed conditions of internal symmetry within the CCF group. Consequently, the size of the whole CCF group need not be explicitly denoted as part of the variable notation for Psg entity in the case that it is evident from the context what is meant by the subgroups. In the connections where the size of the whole CCF group is needed to be emphasized, notation Psg(m|n) can be used. Compare to further discussion of the different probability entities in Chapter 4.

Probability Psg(m) is monotonously decreasing for increasing failure multiplicity. The term dependence profile will be used to mean the shape of Psg-curve, especially related to how failure probability is saturating at high orders.

2.2 Stress-resistance expression

In the CLM, the failure condition is expressed by stress-resistance analogy: at the demand, the components are loaded by a common stress S, and their failure is described by component resistances (strengths) R_i, compare to Figure 2-2 and Eq.(2.3.a). Both the common stress and component resistances are assumed stochastic, distributed variables. A multiple failure occurs, when the load exceeds several component resistances, compare to Eq.(2.3.b) presented as part of Figure 2-2.

In this model, the dependence arises partly from the common load, partly from the identical resistance distributions of the components. However, within the resistance distribution, a component may individually vary from the others. The actual resistances are not exactly known prior to the demand, which effectively places the components in symmetric position (assumption of homogeneity). A relatively wide distribution of resistances means low dependence in general, because then it is less likely that a particular load exceeds several components' resistances at the same. The opposite condition of relatively narrow distribution of resistances means high dependence, because then it is likely that if the load exceeds the resistance of one component the same can happen for several other components also. The properties of the inherent dependence mechanism in CLM are discussed more thoroughly in the original introduction to the model [CLM_77].

The probabilities Psg(m) are the measurable entities: a specific multiple failure profile can evidently be produced by many different choices of the stress and resistance distributions. The normal distributions are still used in the extension ECLM. An extreme load part is added to describe dependencies at high failure multiplicity. In the extension, there are four parameters which are defined and discussed in more detail in Chapter 3 and Appendix 1. Nevertheless, only four parameters prove to describe adequately dependencies in highly redundant, internally homogeneous groups as far as statistical data are available.

STRESS-RESISTANCE EXPRESSION

$S > R_i$ for each i in a specific subgroup of m components (2.3.a)

SUBGROUP FAILURE PROBABILITY EXPRESSION

$$Psg(m|n) = \int_{x=-\infty}^{+\infty} dx . f_S(x) . [F_R(x)]^m \qquad (2.3.b)$$

where

$f_S(x)$ = Probability density function of the common stress

$F_R(x)$ = Cumulative probability distribution of the component resistances

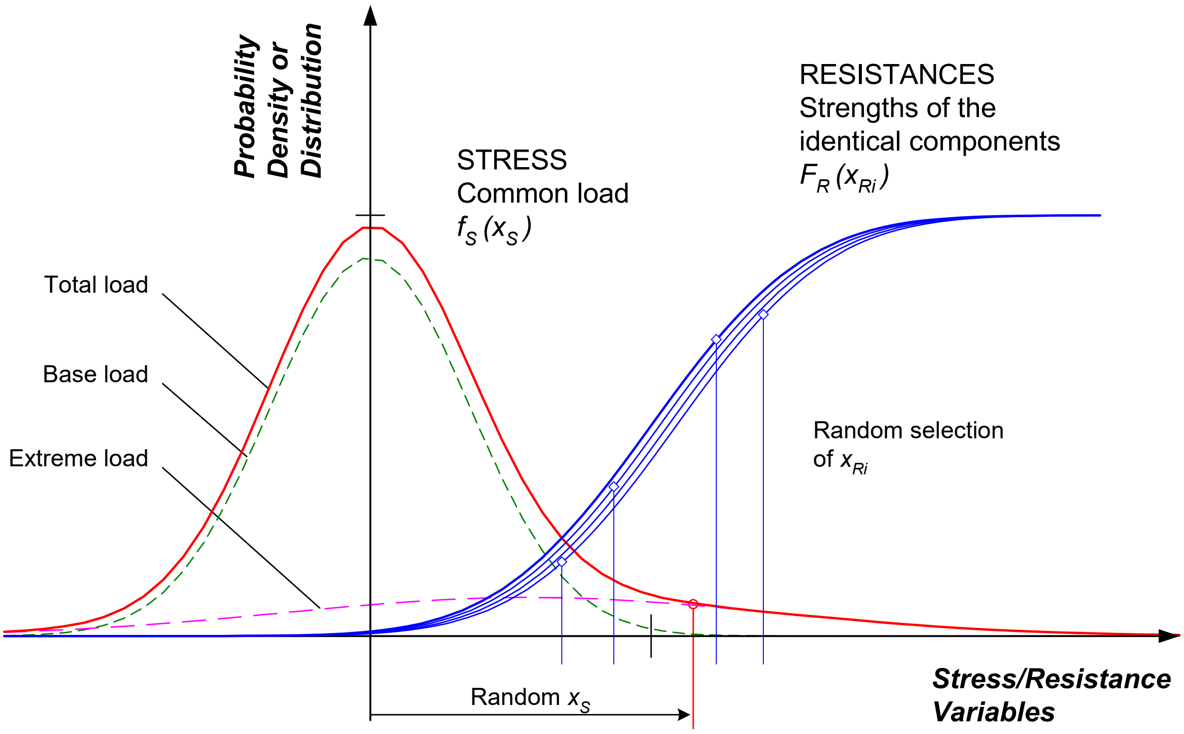


Figure 2-2 Basic concepts in the Extended Common Load Model (ECLM).

2.3 Practical interpretation of the model

The general stress-resistance expression (2.3) can be refined to describe common and independent failure causes for each component in the following way

$$s_C + s_i > r_C + r_i \qquad (2.4)$$

where

s_C = Common stress imposed to all components
s_i = Independent stress fluctuations from component to component
r_C = Common or nominal resistance of the components
r_i = Resistance variations from component to component

Each of these two common entities and 2*n component specific entities are stochastic variables. Assuming the components are identical and form a homogenous CCF group, the s_i have identical distribution and r_i respectively. The expression can be rearranged by moving common entities on the left and component specific entities on the right hand side:

$$\underbrace{s_C - r_C}_{} > \underbrace{r_i - s_i}_{} \qquad (2.5)$$
$$\Leftrightarrow S > R_i$$

Comparing this reduced form to the general stress-resistance inequality (2.3) reveals that the "generalized" common load S comprises those failure mechanism factors that increase common part of stress or decrease common part of resistance. Similarly, the "generalized" resistances R_i comprise those factors which increase individual part of the resistances or decrease individual part of the stresses.

The above discussion applies to a CCF group which is internally symmetric. In case of asymmetry, the stress/resistance variables can be arranged correspondingly, which provides a basis to further extensions of the approach.

If there would be available sufficient knowledge about the failure mechanisms, and statistical distributions for the contributing factors, the analysis could be made "explicitly" at the level of inequality (2.4). In case of some simple structural elements this might be possible in practice. In most cases there exist too large a number of potentially significant failure mechanisms, each too rare to obtain statistical data. Then it is motivated to apply the stress-resistance inequality in its reduced form (2.2), and use some suitable distributions with convenient parametrization. The distributions are fitted to the measurable probability entities Psg(m), in order to estimate the distribution parameters or the equivalent model parameters.

3 Parametrization of ECLM

Due to the way of definition, ECLM is a very general model. The same probability pattern of multiple failures can be obtained (at least with reasonable accuracy) by using many alternative distributions for common load and component resistances. For practical uses it is essential to adopt parametric distributions, which are both well understandable in practical terms, and mathematically convenient to apply.

3.1 The use of normal distributions

In the early definition, normal distributions were used, and simple parametrization with just two parameters, total component failure probability and correlation coefficient, were found suitable for low redundant cases [CLM_77]. In highly redundant cases this proved not sufficient to describe observed probability patterns of multiple failures, i.e. fit to empirical values of Psg(m). Several alternative ways of extension were explored, including the use of extreme value distributions for common load. The trial and error phase resulted in the most suitable option, where load distribution was extended to the superposition of two normal distributions, base and extreme load parts:

$$f_S(x) = w_{Sb}.f_{Sb}(x) + w_{Sx}.f_{Sx}(x) \,, \qquad (3.1)$$

where w_{Sb} and w_{Sx} are weight fractions (positive numbers which sum up to 1). The resistance distribution was retained as sole normal distribution.

Correspondingly, the integration equation, Eq.(2.3.b) divides up into base load and extreme load parts:

$$\begin{aligned} Psg(m) &= w_{Sb}.\int_{x=-\infty}^{+\infty} dx.f_{Sb}(x).[F_R(x)]^m + w_{Sx}.\int_{x=-\infty}^{+\infty} dx.f_{Sx}(x).[F_R(x)]^m \\ &= Psg_{bas}(m) + Psg_{xtr}(m) \end{aligned} \qquad (3.2)$$

The key idea in the extension is targeting the base load part to describe failure probabilities at low multiplicities, and the extreme load part to the (often stronger) dependence at high multiplicities.

The normal distribution is not only convenient to use, but it can be expected to be rather generally valid as in most cases the failure mechanisms are random processes with large number of contributing terms. If the contributions influence additively, normal distribution often applies. On the other hand, if the most significant variables contribute multiplicatively, then lognormal distribution often applies, or alternatively stated, logarithmic transformation brings the situation back to the additive case. The lognormal and normal distributions are fully compatible to be used for underlying distributions of CLM, as shown in detail in [CLM_77].

In the superposition above, the extreme load part can be interpreted to describe environmental shocks, latent design faults, systematic maintenance errors or combinations of these which may cause a large number of identical components to fail concurrently. Basically, further parts can be added to the load distribution. However, two parts have been up to now sufficient to fit with empirically observed probability patterns of multiple failures (in homogeneous groups). The fit has been in fact surprisingly good even for higher order failures in many cases when statistics have been available. Experiences from model fit to failure data are discussed further in Chapter 5. Specific kinds of further extensions have been made in non-homogeneous cases to model internal asymmetry of component group, to be discussed later.

3.2 Choice of model parameters

In the extension, the following four model parameters are defined:

Table 3.1 Parameters of the Extended Common Load Model, range in practical applications.

Parameter	Description	Range	Typical value
p_tot	Total component failure probability	[0, 0.5]	10^{-4} - 10^{-2}
p_xtr	Extreme load part as contribution to the single failure probability	[0, p_tot]	p_xtr/p_tot ≅ 1% … 5% and p_xtr >10^{-5}
c_co	Correlation coefficient of the base load part	[0, c_cx]	0.1 … 0.5
c_cx	Correlation coefficient of the extreme load part	[c_co, 1]	0.6 … 0.9

Mathematical details of the parametrization will be handled in Appendix 1. To summarize, the distributions are scaled as (0, d_{Sb})-normal for base load, (1 - d_R, d_{Sx})-normal for extreme load and (1, d_R)-normal for resistance. The probability parameters are related to Psg(1) and correlation coefficients to standard deviations $\{d_{Sb}, d_{Sx}, d_R\}$ in the following manner:

$$\begin{aligned} p\_tot &= Psg(1); & c\_co &= \frac{d_{Sb}^2}{d_{Sb}^2 + d_R^2} \\ p\_xtr &= Psg_{xtr}(1); & c\_cx &= \frac{d_{Sx}^2}{d_{Sx}^2 + d_R^2} \end{aligned} \qquad (3.3)$$

The parameters form two pairs. The first pair {p_tot, c_co} is related to base load part, and the other pair {p_xtr, c_cx} to extreme load part. The adopted parametrization yields in reduced coupling between the pairs, giving the benefit of intuitively clear impact of the parameters on the multiple failures: pair {p_tot, c_co} describes the probability level and dependence at low multiplicity and pair {p_xtr, c_cx} correspondingly at high multiplicity.

The existence of three underlying distributions $\{f_{Sb}, f_{Sx}, f_R\}$ means in total 6 distribution parameters. Because a linear translation of the stress/resistance variable does not affect the failure probability Psg(k), the degrees of freedom is effectively 5. In the chosen parametrization, one degree of freedom is frozen by anchoring the relative position of f_{Sx} at a specific point between medians of f_{Sb} and f_R, i.e. leaving only the variance of f_{Sx} free. (Base load median is set at 0, extreme load median at 1, and resistance median at 1 - d_R, compare to the details in Appendix 1.) This specific anchoring gives the desired reduced coupling between parameter pairs {p_tot, c_co} and {p_xtr, c_cx}.

It must be emphasized that certain weak coupling is imposed by adopting total component failure probability p_tot as one model parameter. This is due to following relationship, compare to Eqs.(3.2-3):

$$\text{p\_tot} = \text{p\_bas} + \text{p\_xtr}, \text{ where p\_bas} = \text{Psg}_{Bas}(1) \qquad (3.4)$$

Especially, it is good to be aware that in a sensitivity analysis where, for example, p_xtr is varied while keeping other parameters constant (p_tot also constant) base load part adapts in opposite direction compared to p_xtr. Even though the reflected changes in base load part are small they show up clearly in dependence profile in a way that looks strange if one is not aware of the underlying relationship. The influences become simpler and more intuitive by keeping p_bas as constant when varying dependence parameters p_xtr, c_co and c_cx, i.e. allow p_tot floating.

The optional choice of parameter pairs {p_bas, c_co} and {p_xtr, c_cx} as model parameters would be mathematically convenient. But from practical point of view p_bas has to be replaced by p_tot. It should be noticed that p_bas is usually numerically very close to p_tot.

A further coupling is imposed by using d_R in the definition of both c_co and c_cx in order to obtain unified normalization, see Eq.(3.3). These relationships will be discussed in more detail in Appendix 1, Section 8, including advices for the parameter variations in sensitivity analyses.

It needs to be also emphasized that parameters c_co and p_xtr interfere because they contribute in parallel to low order failures. This interference will be discussed closer in the connection to parameter estimation in Section 5.

Usually p_xtr lies at or above the level of 10^{-5} or near to a few percent relative to p_tot, compare to Table 3.1. Mathematically p_tot is limited in ECLM below 0.5 due to the fixed relative placement of distributions. In practice total component failure probability is well below that limit. The correlation coefficients have values between [0, 1]. The value 0 means total independence and value 1 total dependence. Practically meaningful relationship is c_cx < c_co. Often the base load correlation lies in the range of 10%..50%, while the typical value of the extreme load correlation is in the range of 60%..90%.

It should be noted that the parameters defined for the ECLM above, are related to the output of the model rather than to the stress and resistance distributions directly. Only the total component failure probability can be estimated in direct way from the number of failures per number of test/demand cycles. The other parameter estimates cannot be expressed in closed form but must be solved using Maximum Likelihood search or Bayesian method, and computerized estimation tool. The details of estimation will be handled in Chapter 5.

3.3 Subgroup invariance

With the given model parameters, numerical integration of the stress-resistance expression produces probability values for multiple failures, primarily for Psg(m). From the Psg entities then other types of probability entities can be derived following the transformation scheme to be discussed in Chapter 4.

Due to this scheme, ECLM fulfils the subgroup invariance requirement, which is evident also from the definition of the model. This means first of all, that the same model parameters apply in each subgroup, which is very convenient for practical uses. On the other hand, the parameters of CCF groups with different size are directly comparable. This enhances much the utilization of information from analogical cases - without a need to manipulate the event data from different sizes of CCF groups by use of complex mapping up/down methods. Compare to the further discussion of this aspect in [NAFCS-PR03].

4 Calculation of Failure Probabilities

One of the central innovations in this integral approach to the treatment of dependencies in highly redundant structures is the consideration of various kinds of probability entities for subgroup failure, each providing a different point of view. Acronym SGFP is used here for subgroup failure probability.

4.1 Subgroup failure probability concepts

As stated in Chapter 2, we are considering a CCF group of n identical components. The basic entity is the failure probability a specific set of components defined in Eq.(2.1). It will again be emphasized that this entity has following invariance properties in internally homogeneous CCF group (of n components):

$$Psg(k|m) = Psg(k|n) = Psg(k), \text{ for any } k \leq m \leq n, \text{ and} \qquad (4.1)$$

for any selection of k and m components

Subgroup of m components can denote a demand subgroup or any other embedded CCF group to be considered for analysis aims.

Subgroup invariance means that, for example, the probability of k specific components failing is same whether they are considered alone or as a subgroup of the total n components, in both cases, disregarding the information on the other's survival or failure, i.e. $Psg(k|k) = Psg(k|n) = Psg(k)$.

The failure dependencies are in ECLM primarily modelled via Psg entities and numerically derived from integration Eq.(2.3.b). The other (structural) probability entities are then constructed in the following way.

Assuming the demand is imposed on m components, the success criterion can be interpreted also via the equivalent failure criterion k out of m. The associated total failure probability of the reliability structure is denoted by (notation “Pts” comes from total for the structure):

$$Pts(k|m) = P\{\text{k or more out of m components fail} \mid \text{Demand on subgroup m}\} \qquad (4.2)$$

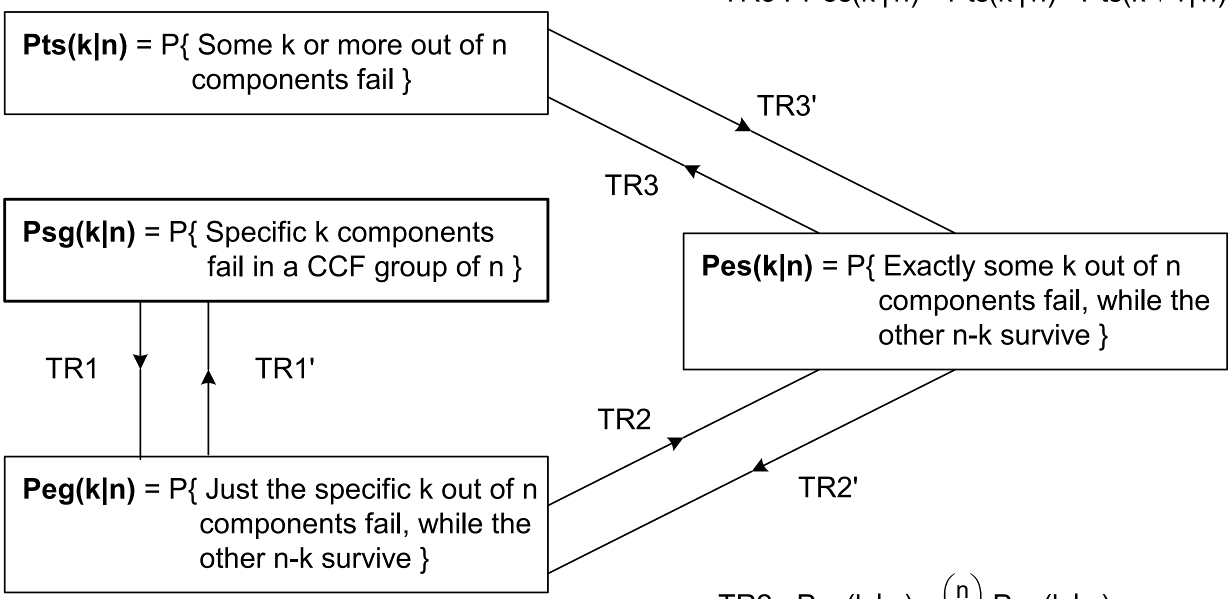
For example, in case of Figure 2-1, the failure criterion is 5 out of 8, and Pts(5|8) is the desired result value.

There is an exact one-to-one correspondence between Pts(k|m) and Psg entities, but Pts(k|m) is not subgroup invariant by any means, instead it is directly coupled to the demand group size m and failure criterion k out of m. The transformations are presented in Figure 4-1 (notice that the equations there are derived for a demand group of size n, but they apply equally well to any subgroup m ≤ n, when n is a homogeneous CCF group). The algorithms are based on standard probability calculus and their derivation is discussed in [SKI TR-91:6].

Transformation scheme
of Subgroup Failure Probability Entities

$$TR3: \; Pts(k\,|\,n) = \sum_{m=k}^{n} Pes(m\,|\,n)$$

$$TR3': \; Pes(k\,|\,n) = Pts(k\,|\,n) - Pts(k+1\,|\,n)$$



$$TR2: \; Pes(k\,|\,n) = \binom{n}{k} . Peg(k\,|\,n)$$

$$TR1: Psg(k\,|\,n) = \sum_{m=k}^{n} \binom{n-k}{m-k} . Peg(m\,|\,n)$$

$$TR2': \; Peg(k\,|\,n) = \frac{1}{\binom{n}{k}} . Pes(k\,|\,n)$$

$$TR1': Peg(k\,|\,n) = \sum_{m=k}^{n} (-1)^{m-k} . \binom{n-k}{m-k} . Psg(m\,|\,n)$$

Figure 4-1 Transformation scheme of subgroup failure probability entities in a homogeneous CCF group of n components. These algorithms apply also within any subgroup of the whole CCF group.

The transformations can conveniently be made by the use of the exclusive subgroup failure probabilities by (notation "Peg" comes from exclusive for the group):

Peg(k|m) = P{Just the specific k out of m components fail, while the other m-k survive|Demand on subgroup m } (4.3)

In some context it is convenient to include the number of combinations of k out of m, into this exclusive probability concept yielding a related entity by (notation "Pes" comes from exclusive for some):

Pes(k|m) = P{Just some k out of m components fail, while the other m-k survive|Demand on subgroup m } (4.4)

If written out, the transformation equations become quite long in case of, for example m=10. But computationally they do not present any big problem as the required code fragments become short when applying usual recursive technique.

In conclusion, Psg entity has a key position among the four different SGFP entities, basically owing to its subgroup invariance. The great benefit of using Psg entity, for example in data comparison, is the fact that it describes the dependence profile of the increasing failure multiplicity without "disturbance" of combinatorics and order exclusion which affect the other SGFP entities. On the other hand, the three other SGFP entities have each a specific area of use in modelling and quantification of dependencies.

4.2 Calculation of SGFPs by the use of ECLM, EPV example

To summarize what was said in the preceding section, the usual flow of calculations is following:

ECLM(Parameters:= p_tot, p_xtr, c_co, c_cx)

→ Psg(k) → Peg(k|m) → Pes(k|m) → Pts(k|m) (4.5)

The calculation order needs to be changed in ultra-high redundant systems because of numerical accuracy limitations with Psg(k) → Peg(k|m) transformation in large groups as will be discussed in Chapter 6. The above calculation order is used in so called 'base' implementation with scope of groups up to 20 … 30 components, depending on ECLM parameter values.

Using the EPV example of Figure 2-1, the result of ECLM integration are first presented in Figure 4-2 and then the SGFP entities are derived for different subgroups in Figure 4-3. Details related to the parameter estimation will be discussed later in Chapter 5. Concerning the demand group of 8 EPVs challenged in a depressurization need, the desired result is Pts(5|8) for failure criterion 5 out of 8.

The example shows a usual dependence profile. Values of Psg(k) saturate quite strongly at higher multiplicity, as the extreme load part Psg_{xtr} becomes then dominating, while the base load part Psg_{bas} is determining at low multiplicities. Due to the strong dependence at higher multiplicities, the Peg(k|m) values become rather small at intermediate multiplicities for bigger subgroups, which is also quite usual.

HiDep Version 2.1
Extended Common Load Model
Avaplan Oy, March 1997

TVO 1/2, electromagnetic pilot valves, best estimate

						Point estimate	
CCF group size		*CLM parameters*				ND	34
KmMax	10	p_tot	4.0E-2	c_co	0.40	VfSum	12.95
		p_xtr	3.0E-3	c_cx	0.80	p_est	3.81E-2

Km	Psg_b	Psg_x	Psg	Zk	Peg	Pes	Pts	Vk	Sk/ND
0	0.991	9.03E-3	1.000	-	0.775	0.775	1.000	26.50	1.000
1	3.70E-2	3.00E-3	4.00E-2	0.040	1.38E-2	1.38E-1	2.25E-1	5.00	2.21E-1
2	6.04E-3	2.15E-3	8.19E-3	0.205	1.04E-3	4.69E-2	8.73E-2	1.00	7.35E-2
3	1.78E-3	1.77E-3	3.55E-3	0.434	1.66E-4	2.00E-2	4.04E-2	0.80	4.41E-2
4	7.15E-4	1.55E-3	2.26E-3	0.637	4.49E-5	9.43E-3	2.04E-2	0.50	2.06E-2
5	3.47E-4	1.40E-3	1.74E-3	0.770	1.87E-5	4.71E-3	1.10E-2		5.88E-3
6	1.92E-4	1.28E-3	1.47E-3	0.846	1.17E-5	2.45E-3	6.29E-3		5.88E-3
7	1.16E-4	1.20E-3	1.31E-3	0.890	1.11E-5	1.34E-3	3.84E-3	0.15	5.88E-3
8	7.47E-5	1.13E-3	1.20E-3	0.915	1.81E-5	8.12E-4	2.50E-3		1.47E-3
9	5.09E-5	1.07E-3	1.12E-3	0.932	6.37E-5	6.37E-4	1.69E-3		1.47E-3
10	3.61E-5	1.02E-3	1.06E-3	0.943	1.06E-3	1.06E-3	1.06E-3	0.05	1.47E-3
							LogLikeL	-21.313	
							DeltaLL	0.000	

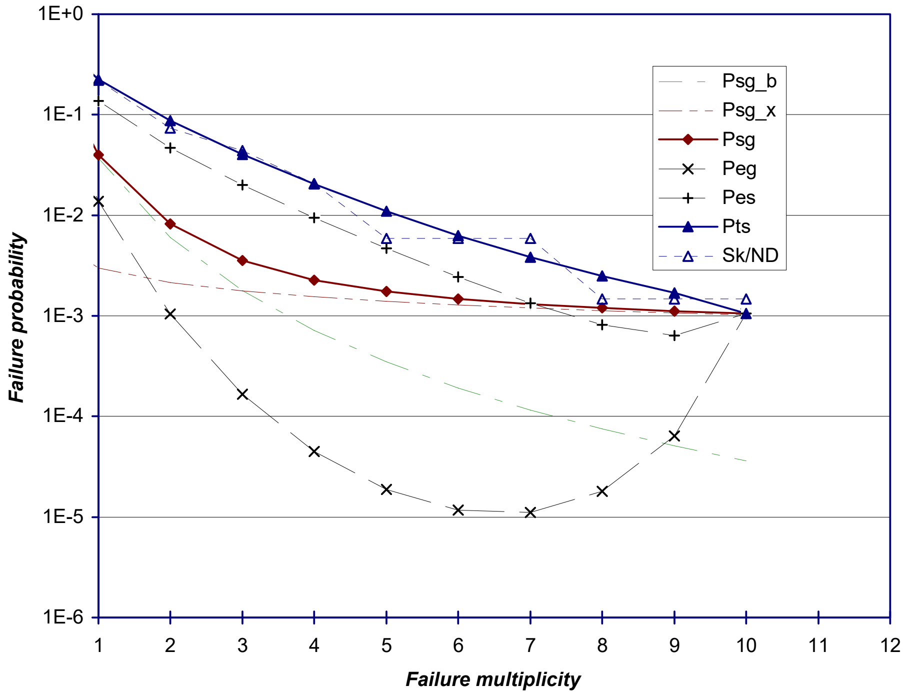


Figure 4-2 Probability quantifications by the use of ECLM for the CCF group of ten EPVs for Olkiluoto 1 and 2 [RESS_HiD].

pts(k|m)

k	pts(k|10)	pts(k|9)	pts(k|8)	pts(k|7)	pts(k|6)	pts(k|5)	pts(k|4)	pts(k|3)	pts(k|2)
0	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
1	1.75E-01	1.61E-01	1.48E-01	1.33E-01	1.17E-01	1.01E-01	8.37E-02	6.52E-02	4.54E-02
2	4.20E-02	3.61E-02	3.03E-02	2.47E-02	1.93E-02	1.43E-02	9.72E-03	5.75E-03	2.55E-03
3	1.24E-02	9.98E-03	7.82E-03	5.90E-03	4.24E-03	2.87E-03	1.78E-03	9.48E-04	
4	4.44E-03	3.50E-03	2.70E-03	2.03E-03	1.49E-03	1.06E-03	6.70E-04		
5	2.09E-03	1.69E-03	**1.37E-03**	1.09E-03	8.43E-04	5.72E-04			
6	1.30E-03	1.11E-03	9.26E-04	7.44E-04	5.18E-04				
7	9.74E-04	8.36E-04	6.83E-04	4.80E-04					
8	7.77E-04	6.40E-04	4.51E-04						
9	6.05E-04	4.28E-04							
10	4.08E-04								

TVO I-II Electromagnetic pilot valves/FO best estimate

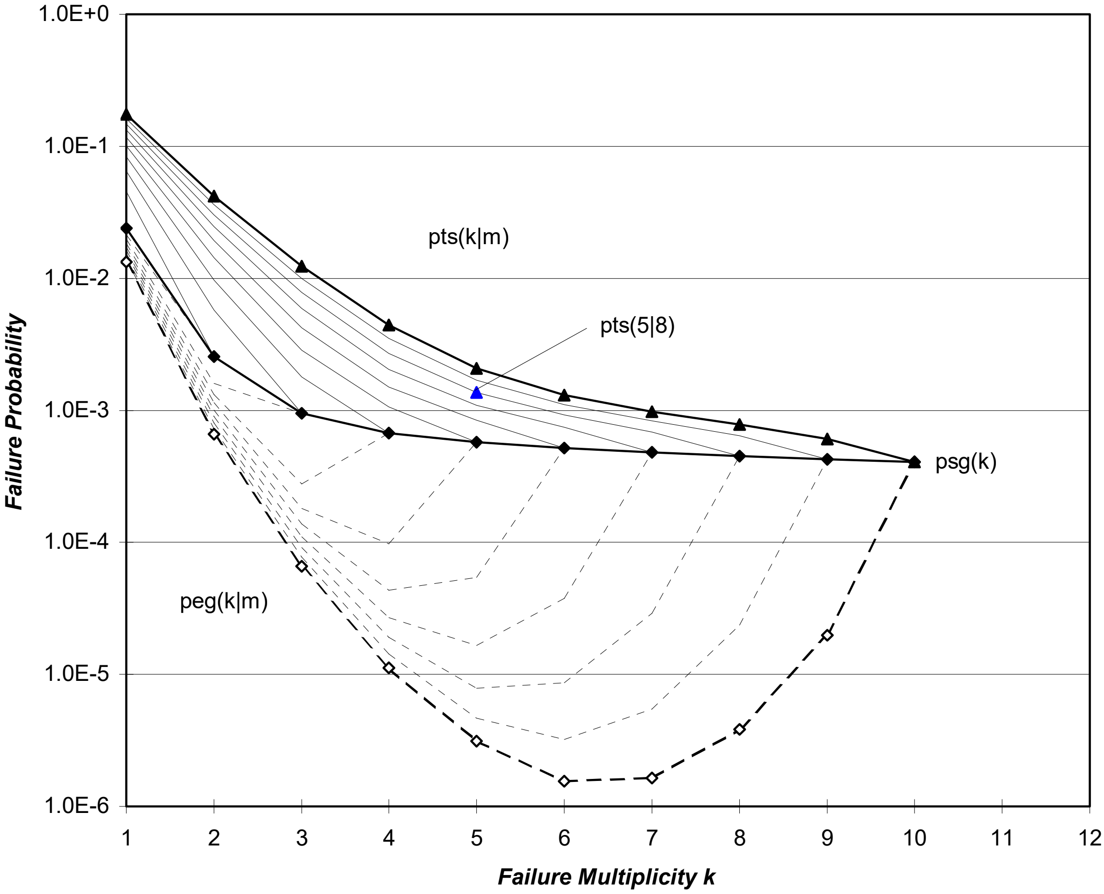


Figure 4-3 Generated failure probabilities for any subgroup of the ten EPVs for Olkiluoto 1 and 2 [SKI TR-91:6].

4.3 Interface to PSA models

In standard PSA approach, the CCF basic events are used as an interface to PSA fault trees. This frame becomes increasingly tedious to use due to rapidly escalating number of event combinations for CCF groups of five or more components. Instead, the dominant contributors can be expressed by a limited number of appropriately defined functional events and by using SGFP entities. For example, in Olkiluoto 1 and 2 PRA (initial design), only 11 interface entities were used for the SRVs as compared to about $2^{10}*2^{2}*2^{12}$ component event combinations between electromagnetic pilots, pneumatic pilots and main valves within 12 SRV modules (10 ordinary safety/relief lines and 2 regulating relief lines).

It should be emphasized, that the definition of interfacing events needs to be done carefully, in order to properly take into account risk-significant cross combinations with loss of activation signals, failure of power buses and other hardware or functional dependencies. Experiences this far show that a rather limited number of these cross combinations are important, which implies that the interface from a highly redundant CCF group can be managed.

As an example, the interface with electric power supply dependence can often be most conveniently structured according to the failure situations of power buses. For example, in the case of depressurization demand, when 8 SRV lines are challenged, and with failure criterion 5 out of 8, the EPVs contribute effectively in the following schematic way (compare to Figure 2-1):

$$Pts_{Eff}(5|8) \cong Pts_{EPV}(5|8) + 4 \, . \, u_{DCB} \, . \, Pts_{EPV}(3|6) \qquad (4.6)$$

where

u_{DCB} = Mean fractional downtime of a DC bus supplying EPVs

This is a usual Rare Event Approximation: the first term corresponds to the situation where the four DC buses are all available, but five or more of the EPVs fail; the second term corresponds to the situation where one DC bus is down for maintenance, and in conjunction three or more of the six EPVs fail in the other subs. In practice, there may exist other cross-combination terms, which need to be included in a similar way [SKI TR-91:6]. The identified terms can then be transformed into equivalent functional event presentation to be incorporated in the system fault trees.

4.4 Comparison with the approaches based on the structure of common cause events

The mostly used CCF models Multiple Greek Letter Method (MGLM) and Alfa Factor Method (AFM) are based on describing dependencies by the use of CCF basic events. Normally, CCF basic events with input probabilities $Q_k^{(n)}$, are explicitly included in PSA fault tree models, and MCS reduction is cared by a computer program. This provides a very convenient interface into PSA models, which works well in low redundancy cases. In high redundancy cases, due to the large number of CCF basic events to be added for many component gates, MCS reduction becomes overwhelmingly burdened. Hence, the pre-processed, reduced presentation in terms of functional events and SGFPs for the PSA model interface as described in the preceding section is the preferred approach.

In practice, if AFM/MGLM would be used, a reduction into the SGFP scheme is the only viable way in highly redundant cases. The problem is that the SGFPs cannot be expressed exactly in terms of the CCF basic events. In low order cases, relatively simple approximations exist, but in high redundancy cases, handling of combinatorics and necessary approximations become cumbersome. Besides, the AFM/MGLM parameters are not subgroup invariant (an example about the parameter variability is presented in [SKI TR-91:6]). This means that cases, where only part of the system is challenged, become laborious to handle. One way is to consider the subgroup as imbedded in the whole group, preserving all CCF basic events for the total group, selecting those which validate the failure criterion in the subgroup demand case. This has been the usual way. Alternative way is to map AFM/MGLM parameters and CCF event probabilities down to the CCF group corresponding with the demanded subgroup. This necessitates the use of rather complicated routines. In contrast, when a subgroup invariant model is used, the same CCF parameters apply to any subgroup.

5 Parameter Estimation

Due to the lack of a reverse analytic solution for the stress-resistance analogy expression, a formula-based direct estimation of ECLM parameters is impossible (except for one parameter, the total component failure probability p_tot). Hence, standard numerical methods for Maximum Likelihood search or Bayesian estimation have to be applied. It should be noticed that most other parametric CCF models, when used in highly redundant cases, necessitate also the use of computerized estimation tools.

5.1 Likelihood Function

The Likelihood Function can be constructed in the usual way given the available information about success/failure events in a group of n components over a number of test/demand events [Henley&Kumamoto]:

$$Lik(p\_tot, p\_xtr, c\_co, c\_cx \mid \{V(k \mid n)\}) = \prod_{k=0}^{n} [peg(k \mid n)]^{V(k|n)} \quad (5.1)$$

where

V(k|n) = the number of failure events of multiplicity k in a CCF group of n components, i.e. Sum Impact Vector

V(0|n) = the number of success events, i.e. test/demand cycles in which all components survived

ND = the number of tests and demands on the whole group of n components

$$= \sum_{k=0}^{n} V(k \mid n)$$

An example of Sum Impact Vector V(k|n) is given in the subtable of Figure 4-2; its derivation from operating experience is explained in [SKI TR-91:6]. Constructing the Sum Impact Vector from the failure records follows the general principles presented in [CCF_PG]. Compare also to the more recent method description [NAFCS-PR03]. A refined approach is developed in an application, which uses state model to describe the development of latent CCF mechanisms and chances to detect them in random demands between test points [T314_TrC].

For the calculation of the Likelihood Function, Peg(k|n) entities are obtained from Psg(k) entities, using the transformation presented in Figure 4-1, while Psg(k) values have first been integrated from the stress-resistance analogy expression, Eq.(2.3.b) - with distribution parameters derived from the model parameters. The whole scheme of the estimation cycle is illustrated in Figure 5-1. As described in Chapter 3 the model parameters are intentionally chosen to represent the failure probability levels and strength of dependence, i.e. the outcome of the model. In the background the model parameters are mathematically connected to the stress/strength distributions. The benefit of this parametrization approach is that the behaviour of the model outcome is well predictable with respect to the parameters.

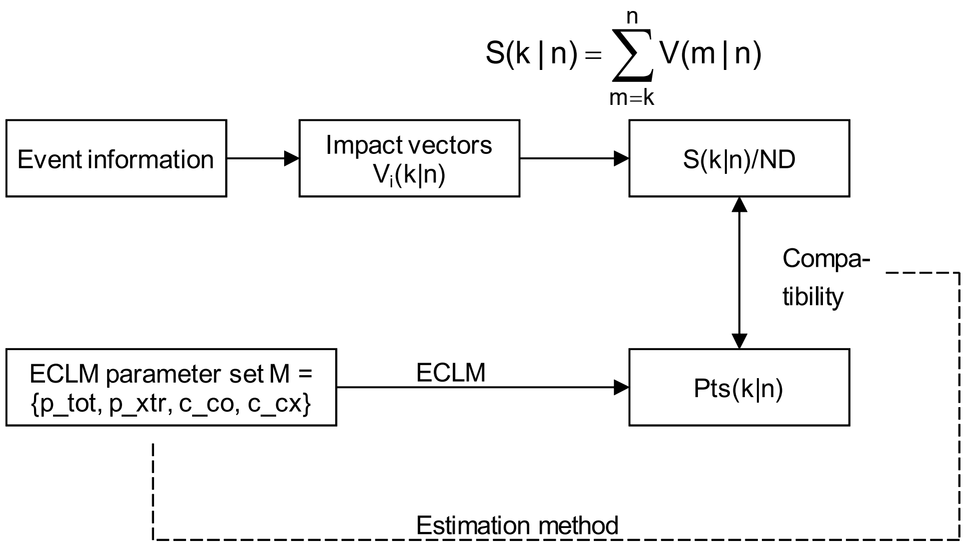


Figure 5-1 Flow scheme of parameter estimation for ECLM.

The drawback is, as said, that the model parameters cannot be directly calculated from the count of failure events. The indirect estimation is required for ECLM. The compatibility verification for the fit of model with the empirical failure pattern will be discussed in more detail in Section 5.3.

Example shapes of the Likelihood Function, with respect to the model parameters p_xtr, c_co and c_cx, are presented in Figure 5-2. The example case is the same as used earlier, the group of ten EPVs in Olkiluoto 1 and 2. Compare to the point estimation in Figure 4-2.

An apparent benefit of Likelihood Function-based approach is that it allows a convenient scheme for combining event data from different sources and from CCF groups of different sizes (which assumes a mutual homogeneity, or a postulation of that as an approximation, see further details of CCF data pooling in [NAFCS-PR03]). In the first approximation, joining data bases A and B results in the following compound Likelihood Function, corresponding to weighting by the total number of test/demand cycles (the Sum Impact Vector without elements is denoted by bold capital letter **V**):

$$\mathrm{Lik}_{A\&B}(.|\mathbf{V}_A, \mathbf{V}_B) = \mathrm{Lik}_A(.|\mathbf{V}_A) * \mathrm{Lik}_B(.|\mathbf{V}_B) \qquad (5.2)$$

Compound Likelihood Function can be used also in the case of asymmetric tests or demands to combine statistical data over test/demand subgroups.

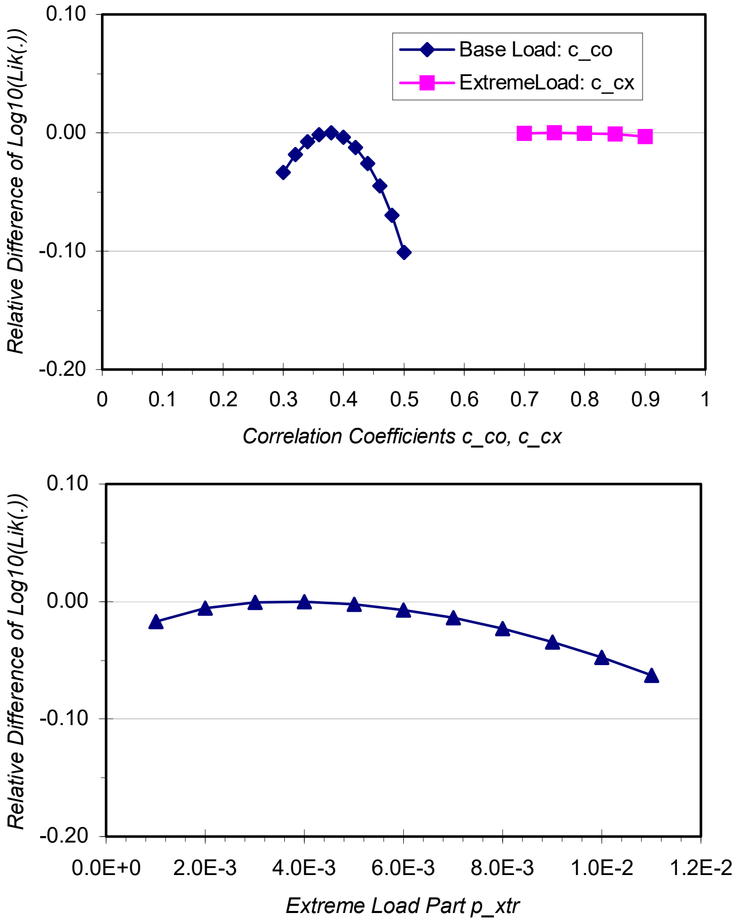


Figure 5-2 Illustration of the Likelihood Function behaviour for p_xtr, c_co and c_cx separately at the maximum [ECLM-Tcases]. The example case is the ten EPVs of Olkiluoto 1 and 2 [RESS_HiD].

5.2 Total component failure probability

The point estimate for the total component failure probability p_tot can be obtained from the total number of failures:

$$p\_tot = \frac{VfSum(n)}{n \cdot ND}, \text{where} \tag{5.3}$$

$$VfSum(n) = \sum_{k=1}^{n} k \cdot V(k \mid n) \ \textit{is} \ \text{the total number of failures}$$

5.3 Comparison with empirical failure pattern

The evidence and the model can best be compared by constructing the empirical failure pattern:

S(k|n) = Number of test/demand events where k or more out of n components are failed

$$= \sum_{m=k}^{n} V(m\,|\,n) \qquad (5.4)$$

Dividing this by the total number of tests/demands ND gives the point estimate of the total structural failure probability, i.e.(look for this kind of comparison in Figure 4-2):

$$\langle Pts(k|n) \rangle = \frac{S(k|n)}{ND} \qquad (5.5)$$

For comparison purposes, this relationship is preferred due to the stable nature of S(k|n) profile even in the case of sparse data. However, the comparison could alternatively be performed between SGFPs and their point estimates:

$$\langle Psg(k) \rangle = \sum_{m=k}^{n} \frac{\binom{n-k}{m-k} \cdot V(m|\,n)}{\binom{n}{m} \cdot ND} \qquad (5.6)$$

5.4 Maximum likelihood search strategy

The Likelihood Function appears to be rather smooth and searching the maximum point can be simplest done by interactively controlled numerical iteration. The point estimate for the total component failure probability p_tot, obtained from Eq.(5.3), can be kept constant during the search process.

The search for the other model parameters can be started from usual values. If no other clue is readily available, the following initial values can be used:

p_xtr ~ 0.03*p_tot (5.7)

c_co ~ 0.4

c_cx ~ 0.8

Practical experiences show, that it is preferable to first search after a reasonable fit of the extreme load part p_xtr, because the dependence pattern is most sensitive for it. Next, continue with c_co and leave c_cx last. In practice, the first iteration round can be based on visual comparison of Pts(k|n) and S(k|n)/ND fit as explained above, compare also to Figure 4-2. Fine tuning can then be performed by considering numerical changes of Likelihood Function. The current version of HiDep Toolbox contains a semi-automated tool for the search of the maximum likelihood estimates.

Because of the stable nature of Likelihood Function, and because the model parameters influence largely independently to outcome probabilities, the maximum likelihood search converges soon for the adopted ECLM parametrization. In most cases less than ten trials are sufficient in order to identify p_xtr, c_co and c_cx all with a reasonable accuracy. The particular details of the parametrization, which broke the dependence between the pairs {p_tot,c_co} and {p_xtr,c_cx} prove thus useful. The model parameters can nevertheless interfere in estimation. For example, c_co and p_xtr can contribute in parallel to low order failures, implying a diagonal correlation in Likelihood Function. The surface determined by Likelihood Function can rise in a direction deviating from c_co and p_xtr coordinate directions. This aspect can be controlled by watching Likelihood Function values for a grid of parameter values, or by using some developed search algorithm for the maximum of a multi-parameter function.

In case of sparse data for high multiplicity failures, extreme load part p_xtr can be considered primary parameter for the dependence at high multiplicity, while extreme load correlation c_cx has a side role, and can be preserved at the typical value of 80%.

For correlation coefficients c_co and c_cx, the Likelihood Function is usually rather symmetric on the linear scale, and the maximum point is apparent, Figure 5-2. The width of the maximum area may be quite broad (absolute value of the second derivative small) if data are sparse.

For the extreme load part p_xtr, it is more natural to consider the Likelihood Function on the logarithmic scale for p_xtr: it is usually rather flat towards small values, but decreases steeply for higher values when approaching the area which contradicts with the evidence about the number of high multiplicity failures, or if p_tot will be approach. If data are sparse, there may not be a maximum point at all for p_xtr, but the Likelihood Function may be monotonously (although weakly) increasing for decreasing p_xtr. If this happens to be the case, the Bayesian estimation approach is to be preferred. In the absence of Bayesian estimation tool, the following shortcut can be used: p_xtr should be chosen at the level where the Likelihood Function begins to significantly reduce from the plain level at smaller values of p_xtr.

5.5 Experiences from parameter estimation

By generic data is meant here typical parameter values estimated for a collection of similar component types, similar with respect to complexity, share of mechanical and electrical parts, component boundary, and operational, test and maintenance environment, as well as similarity in defenses against CCFs.

Some data compilations contain CCF data pooled over much different component types, e.g. pooled data of all demand failures. Such data represent a statistical mixture of failure mechanisms and CCF types which can be problematic to apply to a specific target component. It is hence recommended to find out reference data for earlier analyzed similar component type.

A possible last resort is the use of parameters of so called Generic Dependence Classes, see discussion in [Nafcs-PR02, Section 4.2]. They were originally derived for CCF groups of size up to four, covering component types like pumps, valves, breakers and diesel generators. So they may not be applicable in highly redundant systems.

The statistical input (Impact Vector) can in some cases contain a large number of complete CCFs. This can mean that no sensible parameter estimates are obtained, e.g. maximum likelihood is obtained at null correlation coefficients. This situation can be related to mapping up of complete CCFs observed in smaller component groups, which may be unrealistic conservative. The dilemma of mapping up has been discussed in [NAFCS-PR03, Section 6.2]. Possibly, actual complete CCFs can have actually occurred due to some specific causes in the size of CCF group under consideration. Exceptionally large number of complete CCFs is most likely related to special CCF mechanisms like external events or other events beyond normally defined component boundary, or systematic operational, test or maintenance errors or system design errors, or combination of these. A possible solution is to separate special complete CCFs and model them explicitly, and cover the other part of statistical data by parametric CCF model.

Another peculiar Impact Vector is such a case where the elements are strongly decreasing as the function of multiplicity. This situation can arise, for example, if binomial model is used to map up potential CCFs, compare to the discussion in [NAFC-PR03, Section 6.2]. Maximum of Likelihood Function can be then obtained with null extreme load part. The statistical significance of Impact Vector is in such a case anyhow weak at high orders. Generic data can be used for extreme load part parameters. Leaving extreme load part out reduces ECLM to the early defined two parameter CLM. Experiences thus far show that in all cases with sufficient statistics the dependence profile of real components contains an outstanding extreme load part.

In some cases, the statistical input can show peculiarities caused by inhomogeneity. For example, exceptionally many failures of order two may have occurred in a group of four components due to weaker defence against CCFs between redundancy pairs, i.e. the group is pair-wise asymmetric. A recommended solution is to divide CCFs to those possible for the whole group and those specific to redundancy pairs, resulting in smaller CCF subgroups to be modelled in addition to the whole group.

CCF model comparison [CCFCoFin] contained estimation tests with a large number of different data sets. This material can provide useful insights for the elaboration of new data cases.

5.6 Model behaviour as the function of ECLM parameters

Inherited from the definition, the model parameters have the following meaningful ranges for practical application:

$$0.5 > p\_tot > p\_xtr > 0 \qquad (5.8)$$
$$0 < c\_co < c\_cx < 1$$

Looking for the visual "look and feel", it should be emphasized once more, that the parameters form two pairs. The probability parameters p_tot, p_xtr give the value wherefrom the associated Psg-curves start at multiplicity k=1. The correlation coefficients c_co, c_cx describe the slopes of the curves: small value means weaker dependence and steeper decrease of the probability as the function of failure multiplicity, while a value of c_cx near to 1 would result in strong saturation at higher failure multiplicities.

As a specific feature of this approach, it should be noted that the CLM inherently allows individual variation in component vulnerability to failure, as described by the resistance distribution. As a benefit, this model will not show so strong tendency of producing overly pessimistic dependence at high multiplicity in case of sparse data, as most other models, which often reduce effectively to a conservative "cut-off" probability at high failure multiplicities.

5.7 Bayesian estimation outline

Besides maximum likelihood search, the Likelihood Function serves as a basis for the more developed Bayesian estimation. This approach will be shortly outlined here as related to beginning experiments used to estimate Pts(k|n). According to the standard Bayesian approach, the expected value of model related Pts(k|n) can be obtained from:

$$E[pts(k|n)] = \iiint dp_x.dc_o.dc_x.f_{Post}(p_x, c_o, c_x).pts(k|n; p_x, c_o, c_x) \qquad (5.9)$$

where the normalized, compound posterior density function is

$$f_{Post}(p_x, c_o, c_x) = \frac{f_{p\_xtr}(p_x).f_{c\_co}(c_o).f_{c\_cx}(c_x).Lik(p_x, c_o, c_x|\mathbf{V})}{\iiint dp_x.dc_o.dc_x.f_{p\_xtr}(p_x).f_{c\_co}(c_o).f_{c\_cx}(c_x).Lik(p_x, c_o, c_x|\mathbf{V})}$$

Here, the total component failure probability p_tot is considered fixed (but the scheme could be readily extended to handle it as the fourth free parameter). Apriori distributions are noted as f_{p_xtr}, f_{c_co}, f_{c_cx} for the other three model parameters considered free variables in the estimation. For the Likelihood Function, a shorthand notation is used here as compared with Eq.(5.1). Similarly, the notation for parameters p_xtr, c_co, c_cx are shortened to p_x, c_o, c_x.

For the apriori distributions, the simple piecewise linear- constant shapes are used as shown in Figure 5-3, considering p_xtr on the logarithmic scale and correlation coefficients c_co, c_cx on the linear scale. The distribution shapes reflect the judgment about the

- possible range of parameters (nonzero distribution density)
- most likely range (constant distribution density)

The derived expected values are noted on Figure 5-3. They prove to be in quite a good agreement with the maximum likelihood values, assuming a reasonable amount of data. The behaviour of Likelihood Function for c_cx is not shown here, it is very flat, compare to Figure 5-2. The expected value of c_cx is thus determined in the example case by the a priori distribution. Even generally, the a-priori distributions have a big role and their choice is therefore crucial in Bayesian estimation.

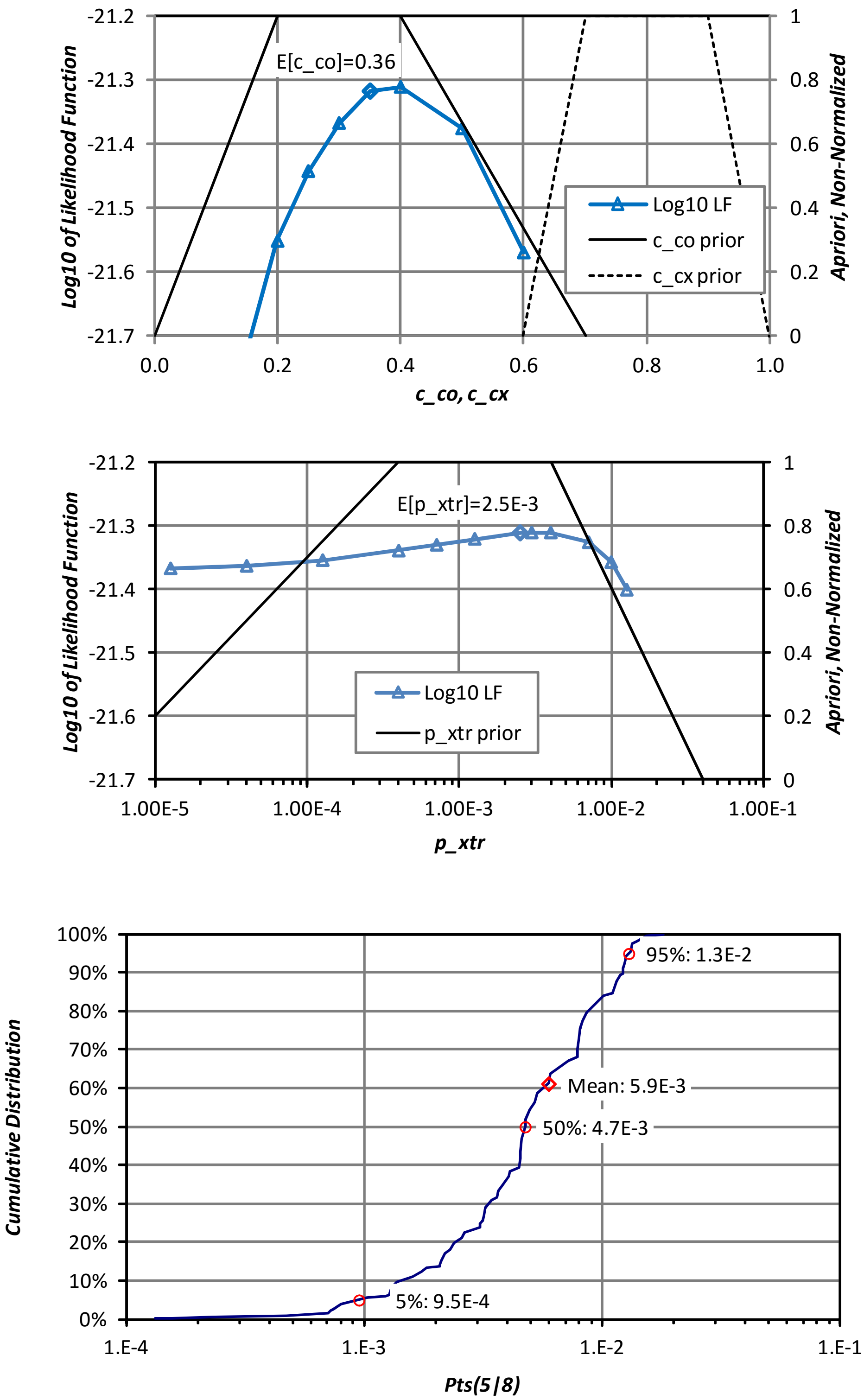


Figure 5-3 Illustration of the behaviour of Likelihood Function, and a priori distributions and posterior distribution of Pts(5|8) in Bayesian estimation experiments in 1990.

5.8 Uncertainty treatment

The uncertainties and their treatment in the CCF analysis of highly redundant systems is similar to probabilistic safety analyses in general. The sparse data on high multiplicity failures mean that the uncertainty range is large. Engineering judgment is often used extensively in the interpretation and extrapolation of the data. A special difficulty is concerned with how to take into account design changes or other countermeasures which are usually implemented after CCF occurrences. The verification of the learning effect is a difficult issue.

The uncertainty impact on the results and conclusions of the analysis can be in the simplest approach be investigated by sensitivity analyses/parameter variations. A more consistent evaluation of uncertainties can be made by Bayesian approach which is straightforward to apply in case of ECLM, representing a well-behaving, ordinary parametric model.

Advanced techniques are recently developed in France for ECLM parameter estimation [PEstim-ECLM-FR2013].

6 Ultra High Redundant Systems

This chapter handles application of ECLM to ultra-high redundant systems like control rods and drives extending the methodology description presented earlier in [SKI Report 2006:05] and recent application reports.

Special care is required for numerical accuracy in the calculation of multiple failure probability at the orders with very high number of component combinations as will be discussed in Section 6.1.

In the case of control rods and drives an additional specific aspect is important. The failures of adjacent control rods impose much bigger reactivity effect in comparison to spread failure placements. This aspect concerns failure consequences but does not violate the assumption about homogeneity of CCF group. The treatment of adjacent rod patterns in the definition of failure criteria will be discussed in Section 6.2.

Some CCF mechanisms are more likely in the case of adjacent control rods and drives, representing a special type of non-homogeneity of CCF group. These so called localized failure mechanisms are considered in Section 6.4. Main part of CCF mechanisms can be regarded to have no position correlation, i.e. control rod and drives can be modelled as a homogeneous CCF group with respect to them. These so called scattered failure mechanisms are considered in Section 6.3, covering also the general case of ultra large homogeneous CCF group. Section 6.5 combines the calculated results over different failure mechanisms.

The failure probabilities are considered in this chapter as so called demand failure probabilities.

6.1 Arrangement of probability calculations in very large groups

The calculations in very large CCF groups have to be arranged so that Peg entity is derived first from the following version of ECLM expression, compare to analogous Eq.(2.3) used to obtain Psg entity:

$$Peg(m|n) = \int_{x=-\infty}^{\infty} dx \cdot f_S(x) \cdot [F_R(x)]^m \cdot [1 - F_R(x)]^{n-m} \quad (6.1)$$

This route is necessary to avoid problems with small differences in Psg entity values and very large binomial coefficients at the intermediate multiplicities in a very large group, implying that transformation Psg(Km) → Peg(Km|KmMax) containing alternative positive/negative terms cannot be handled with a standard double accuracy of computer programs in ultra-high redundant cases. However, the opposite transformation Peg(Km|KmMax) → Psg(Km) contains only positive terms, and is manageable up to CCF group sizes of at least about 160, a practical limit in the applications to control rods and drives of Nordic BWRs.

Alternatively, Psg and Peg entities can be integrated in parallel. The transformations Peg(Km|KmMax) → Pes(Km|KmMax) → Pts(Km|KmMax), needed to find out the failure probability of the system function for a given failure criterion, deal only with positive terms, and is manageable also in ultra-high redundant cases.

An implication of the above feature is that scale down to a subgroup cannot be performed based on a subset of Psg(Km) similarly as in the base implementation of ECLM, using the subgroup invariance property of Psg entity, compare to earlier discussion in Section 3.3. In ultra-high redundant case the subgroup system needs to be re-quantified in order to obtain Peg entity within the subgroup, which effectively means a change of the CCF group size (KmMax), a drawback of 'ultra' implementation. Scale down needs are fortunately not frequent in ultra-high redundant systems whereas they are typical in CCF modelling of safety/relief valves and similar systems.

It has to be noticed that there is no low bound for using ultra-implementation (with the sacrifice that probability entities of subgroups cannot be derived without complete recalculation). Thus ultra-implementation can be applied to a group of twelve safety/relief valves, for example, to verify the results with respect to using base implementation for the same group (with same data).

6.2 Failure criteria for control rods and drives

The placement of control rods with respect to horizontal cross-section of reactor core can be described by so called control rod map where X and Y coordinates are given to each control rod. A selection of control rods can be denoted by the following different ways:

- Group, especially when meaning physical components
- Combination or set, typically in the treatment of failure events
- Pattern in the control rod map, like in Figure 6-1

Group size refers to the number of included control rods. Term shape is used for geometrical type of control rod pattern.

The third dimension, coordinate Z describes the degree of insertion. Usually it is expressed by the degree of withdrawal: fully inserted as Z = 0% and completely withdrawn as Z = 100%. This dimension is not usually taken into account in risk analysis. In normal operation part of control rods is completely withdrawn, part partially inserted. It is conservatively assumed that all control rods are completely withdrawn before demand. Furthermore, partial failure of insertion (stagnation at an intermediate position) is not covered among the modelled failure modes, only complete failure of insertion. The treatment of complex phenomena in failure event analysis and classification is discussed in more detail in the recent data analysis [SKI Report 2006:05].

The failure criterion for control rods is usually expressed by two conditions combined with Boolean OR:

- The pattern of failed control rods contains a critical number and placement of adjacent control rods (K_{Cri}), or
- The number of failed control rods placed in any positions exceeds a threshold number (K_{Thr})

Two control rods are called as tightly adjacent if their X coordinates are same and Y coordinates differ by one step, or vice versa. Diagonally adjacent placement means that both X and Y coordinates differ by one step. The first type imposes much larger reactivity effect. Shapes 3.1-2 in Figure 6-1 constitute of tightly adjacent placements while Shape 3.3 is a mixture of tightly and diagonally adjacent placements. Tightly adjacent pattern is defined as a placement of failed control rods where each control rod is tightly adjacent to at least one other control rod in the pattern.

More specifically, the first condition of failure criterion is defined in terms of Minimal Critical Shapes. Minimal Critical Shape represents basic type of tightly adjacent patterns with critical number of adjacent control rods. Attribute 'minimal' expresses the condition that removing one rod makes the pattern non-critical. Minimal Critical Shape is logically similar to Minimal Cut Set but has the special property of being a two-dimensional geometrical pattern (of adjacent failing rods in the horizontal cross-section of core, i.e. in control rod map). Minimal Critical Shape is also a type collection of geometrically equivalent patterns that can be obtained from a basic one by rotation, as mirror image and/or by linear translation in control rod map. Figure 6-1 illustrates Minimal Critical Shapes that are discussed later in more detail. Different instances of Minimal Critical Shape (placements on core map) constitute Minimal Cut Sets (failure combinations of concerned control rods and drives).

Effects at the core edge are usually truncated, i.e. the presented failure criteria apply across the whole core, and the core is treated in certain sense as infinite even though the number of control rods is finite. It is possible to divide the core into centre and outer zones, defining failure criteria separately in the zones. This option makes the treatment of failure combinations more demanding.

It is preferable to model control rod drive as combined together with control rod, compare to the description of control rod and drive components in [SKI Report 2006:05, Section 2.1]. This super-component, Control Rod and Drive Assembly (CRDA) has dual functions: hydraulic insertion and screw insertion. Correspondingly, it has three functional failure modes: failure of hydraulic insertion (exclusively), failure of screw insertion (exclusively), and failure of both functions. The failure criteria can differ among the failure modes depending on the reactivity shutdown requirements in different transient conditions.

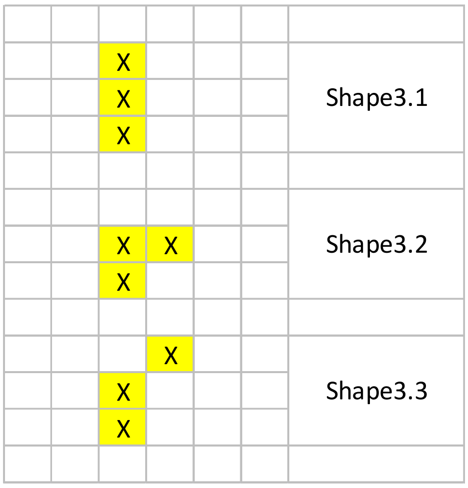


Figure 6-1 Minimal Critical Shapes of the adjacent failing control rods handled in F1/F2 reference application [SKI Report 2006:05, Figure 5-3]. (Minimal Critical Shapes are denoted here by Shape3.1/2/3 corresponding to notation Shape 1/2/3 in the named reference.)

6.3 Scattered failure mechanisms

As defined, scattered failure mechanisms affect in homogeneous way control rods and drives. In a homogeneous CCF group – and without excess criticality imposed by failures of adjacent control rods – the functional failure probability is obtained as Pts(K_{Thr}|KmMax) which can be expressed as

$$Pts(K_{Thr}|KmMax) = \sum_{Km=K_{Thr}}^{KmMax} CmbS(Km|KmMax) \cdot Peg(Km|KmMax) \quad (6.2.a)$$

CmbS entity represents the number of random combinations of Km rods out of KmMax (precisely):

$$CmbS(Km|KmMax) = \binom{KmMax}{Km} \quad (6.2.b)$$

The above probability expression applies to the general case of (ultra large) homogeneous CCF group. In the case of control rod and drives the possible coincidental inclusion of critical patterns of adjacent rods in the random failure combinations needs to be taken into account. A smaller number of randomly placed rods than threshold K_{Thr} can contain a Minimal Critical Shape or other kind of critical dense pattern implying failure of reactivity shutdown.

The number of combinations of Km rods containing a Minimal Critical Shape is approximately

$$CmbA(Km|KmMax) \cong N_A \cdot \binom{KmMax - KmA}{Km - KmA} \quad (6.3)$$

where

KmA = Size of Minimal Critical Shape

N_A	=	Number of different placements of Minimal Critical Shape within core
KmMax	=	Total number of control rods

In order to take into account other kinds of critical dense pattern, so called interpolation rule is used to calculate the weighted number of critical combinations CmbW(Km|KmMax) from the bounds:

Low bound =	CmbA(Km|KmMax)	=	Number of combinations for Km rods containing Minimal Critical Shape A
High bound =	CmbS(Km|KmMax)	=	Number of all random combinations for Km out of KmMax rods

Logarithmic interpolation algorithm will be used to calculate the weighted average:

$$\text{CmbW(Km|KmMax)} = \text{CmbA(Km|KmMax)},\ \text{if Km} \leq \text{Km1} \tag{6.4}$$
$$= \text{Exp}[\text{w1}\cdot\text{Ln(CmbA(Km|KmMax))} + \text{w2}\cdot\text{Ln(CmbS(Km|KmMax))}],\ \text{if Km1} < \text{Km} < \text{Km2, with}$$
$$\text{w1} = (\text{Km2} - \text{Km})/(\text{Km2-Km1}) \text{ and}$$
$$\text{w2} = (\text{Km} - \text{Km1})/(\text{Km2-Km1}) = 1 - \text{w1}$$
$$= \text{CmbS(Km|KmMax)},\ \text{if Km} \geq \text{Km2}$$

Within interpolation range Km1 < Km < Km2 applies CmbA < CmbW < CmbS. The interpolation is linear on logarithmic scale with respect to failure multiplicity.

The weighted average is aimed at taking into account the possible inclusion of other types of dense critical patterns besides of Minimal Critical Shape at increasing order of failure combinations. The start of interpolation range can be set as Km1 = 2·KmA.

The presented formula for CmbA(Km|KmMax) is an overestimation at large multiplicity because of double-counting instances of Minimal Critical Shape. The overestimation is, however, small below break-even point KmB at which the approximation exceeds the number of random combinations CmbS(Km|KmMax). See Figure 6-2 which compares the number of combinations in case of Minimal Critical Shape of 2x2 rods and Barsebäck 1 and 2 core with KmMax=109 [SKI Report 96:77]. It is relatively simple for this regular shape to find out by sampling technique the fraction of random patterns containing it once or several times. The comparison is made on relative logarithmic scale because dealing with huge numbers on absolute scale, e.g. CmbS(40|109 = 1.03E+30. Insights from the applications show advisable to set the end of interpolation range as Km2 ≅ KmB.

Compare to the interpolation range [6, 15] used in F1/F2 reference application [SKI Report 2006:05, Section 5.4.2 and Figure 5-4].

Limited sampling of failed rod patterns was performed in the recent analysis upgrade for Forsmark 1 and 2 [NPSAG 01-04-RADDA]. The reactivity influence of failure patterns was assessed by calculated multiplication factor in selected core conditions. The experiments showed that the treatment of scattered failure mechanisms by interpolation rule is reasonable, seemingly on conservative side.

Count N_A can be derived relatively easily manually by figuring out different placements of Minimal Critical Shapes on core map. For example, in the case of Forsmark 1 and 2 core and three shapes defined in Figure 6-1 it is obtained N_A = 1830, composed from the following contributions for the different shapes:

Shape 3.1: there are 131 different linear placements and 2 different rotations, i.e. 131*2=262 (mirrored image equals to the initial one)

Shape 3.2: there are 136 different linear placements and 4 different rotations, i.e. 136*4=544 (mirrored image equals to the initial one rotated by 180 degrees)

Shape 3.3: there are 128 different linear placements, 4 different rotations and 2 mirrored image of the basic shape, i.e. 128*4*2=1024.

Weighted Pts entity can be obtained from (for Km ≥ KmA):

$$PtsW(Km|KmMax) = \sum_{Ks=Km}^{KmMax} CmbW(Ks|KmMax) \cdot Peg(Ks|KmMax) \quad (6.5)$$

The net result for scattered failure mechanisms is so derived as PtsW(KmA). The second part of failure criterion is thus not directly used in the probability calculation. A remarkable difference between KmB and K_{Thr} indicates that the two parts of failure criterion may not be compatible. In case of K_{Thr} < KmB the end of interpolation range can be conservatively forced to Km2 = K_{Thr}. In the opposite situation with K_{Thr} > KmB it is recommended to set Km2 = KmB. It must be emphasized that the assessment of K_{Thr} is much based on engineering judgment, and is diffuse in the sense that among the random combinations of given number of failed rods an increasing part of combinations is critical as the function of increasing failure multiplicity.

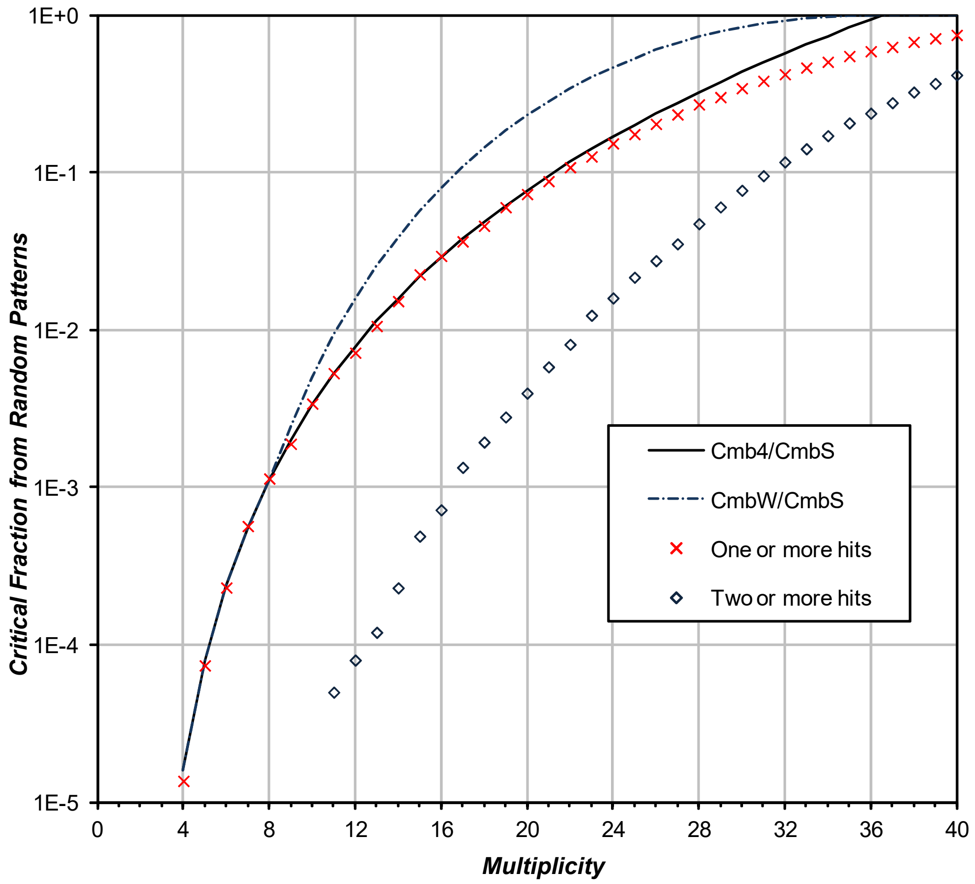


Figure 6-2 Number of combinations in case of Minimal Critical Shape of 2x2 rods and Barsebäck 1 and 2 core containing 109 rods. Sampling results are presented by using following entities/definitions:

One or more hits =	Number of sampled patterns of given multiplicity, containing at least one Minimal Critical Shape of 2x2 adjacent rods
Two or more hits =	Number of sampled patterns of given multiplicity, containing at least two instances of Minimal Critical Shape of 2x2 adjacent rods
Cmb4 =	Number of random patterns of given multiplicity, containing at least one Minimal Critical Shape of 2x2 adjacent rods, analytic approximation
CmbW =	Number of random patterns of given multiplicity, being critical, weighted approximation
CmbS =	Number of all random patterns of given multiplicity, used for normalization of the presented entities

6.4 Localized failure mechanisms

Operational experience indicates CCF types which affect more likely neighbor rods in comparison to rods separated by distance. The position dependence can be of the following two types:

- radial correlation type, failures concentrate symmetrically around some position
- band correlation type, failures concentrate within a band around the core, i.e. at about the same distance from the core centre

The physical background and observations of position dependence are discussed more in [SKI TR 96:77, SKI Report 2006:05].

ECLM implementation assumes that some rod is firstly and/or most severely affected, called as reference rod and denoted as Rod ∅. The rod positions under influence form inner and outer shells, see Figure 6-3. Band correlation is simplified in modelling by two rows of adjacent rod positions, Rod ∅ in the second row. There is an equivalent mirrored configuration where Rod ∅ is in the first row, closer to the core centre if that is in upward direction in the diagram, Figure 6-3. This degree of freedom is implicitly taken into account in the fraction of band correlated type, to be handled in Section 6.5.

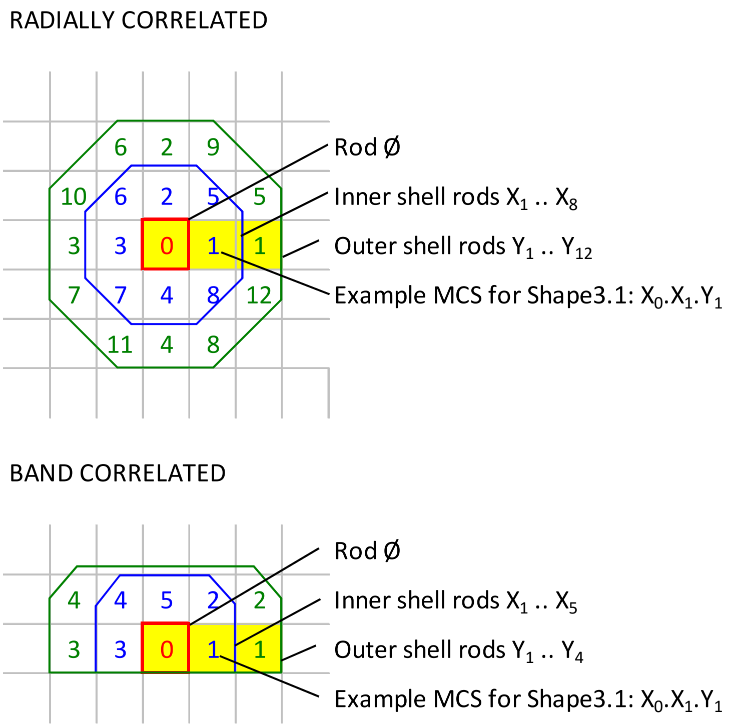


Figure 6-3 Shell layers in modeling of localized CCFs.

The failures of the rods most adjacent to the reference rod are assumed to be stronger dependent and of next adjacent rods weaker dependent. Dependence beyond outer shell (and band) is neglected. In both types the conditional failure probability of neighbour rods given failure of Rod ∅ is dependent on the distance from Rod ∅ in the same manner but in radial type the number of failure combinations is larger. The mathematical details are described in Appendix 2. Main aspects are following:

- It is assumed as if all observed failures would fall into a type of localized failure mechanisms; hence p_tot is set equal to the total component failure probability of CRDA (of a given failure mode) multiplied by the number of CRDAs; the interpretation of p_tot is, in this context, the probability that a localized failure mechanism is present somewhere in the core, one rod being most severely affected
- The different level of dependence is taken into account by different parameter of extreme load part p_xti and p_xto for inner and outer shells, respectively. The lower conditional probability of outer shell components is modelled by a shift of resistance distribution
- Extreme load part parameter p_xto (outer shell) is set equal to the value of p_xtr that has primarily been estimated for randomly scattered failure mechanisms; this corresponds to the judgment that the failure probability of CRDAs, which are separated by distance, should be similar for localized and scattered failure mechanisms
- The ratio of p_xti/p_xto = 10 for localized failure mechanisms is based on judgment; the value of this ratio is less critical than the level of p_xto
- Correlation coefficients have same values for localized failure mechanisms as for scattered failure mechanisms

It is advisable to present the total failure probability in terms of Psg entities, because then the other rods beyond the outer shell are conveniently excluded. This unburdens combinatorial analysis and calculations substantially. It has to be emphasized, that Psg entities are defined as the probability of specific components failing irrespective of the condition of other components beyond outer shell (they may either operate or fail). This leads to the following formula:

$$Pcri = \sum_{Kos} \sum_{Kis} Cmb(Kis, Kos) \cdot Psg(Kis, Kos) \tag{6.6}$$

where

Cmb(Kis, Kos)	=	Combination coefficients
Psg(Kis, Kos)	=	Probability that in addition to Rod ∅, specific Kis inner shell and Kos outer shell rods fail

The array of combination coefficients is derived by Boolean reduction of the failure event expression. The derivation starts from the probability expression in terms of Minimal Cut Sets:

$$Pcri = P\{TOP\} = P\{M_1 + M_2 + \cdots + M_N\} \tag{6.7}$$

where M_i denotes Minimal Cut Sets, i.e. different placements of Minimal Critical Shape completely inside inner and outer shells, including rotated and mirrored images. The exact reduction can be obtained by so called Inclusion-Exclusion Principle [Henley&Kumamoto, p.320]. It is worthwhile to notice that no truncations or other approximations are involved in this stage. The terms in reduced probability expression are arranged with respect to the number of inner and outer shell components, yielding combination matrix. Because Rod ∅ is in every Minimal Cut Set and in every term of reduced probability expression, it is convenient to index by Kis=0 and Kos=0. The reduction process is exemplified in Appendix 2.

Regarding the realization of numeric calculations, see quantified cases in [SKI TR 96:77 and SKI Report 2006:05].

The used definition of inner/outer shell configuration has varied in the course of applications. This is related to sparse information about CCF events which does not give any strong guidance in this aspect. The shell layout of Figure 6-2 has been used in the recent applications where the size of Minimal Critical Shape has varied from 2 to 6. In the latest upgrade for Forsmark 1 and 2 with more optimistic failure criteria, and size of Minimal Critical Shape 9 or 12, the shell layout was somewhat extended [NPSAG 01-04-RADDA].

The introduced extension of CLM proves to behave in a practically consistent way, and because of requiring only one additional model parameter, it is still well manageable. Admittedly, the estimation of model parameters from empirical data is difficult even in the long term because of sparse occurrences of CCFs containing position dependence. Thus, the model should in principle be considered as a sensitivity analysis tool, which relies much on engineering judgment.

6.5 Combining failure mechanisms

The failure mechanisms were divided in following categories with respect to position correlation:

Scatt	Scattered failure mechanisms, no position correlation
LocZ.B/R	Localized failure mechanisms, position correlation of band/radial type

Probability calculations are performed firstly for each category assuming as if it would encompass all failures. The overall result (of a given failure mode) is assembled using assessed fractions of categories, schematically:

$$P_{RS} = w_{Scatt} \cdot PtsW(KmA|KmMax) + w_{LocZ.B} \cdot Pcri_{LocZ.B} + w_{LocZ.R} \cdot Pcri_{LocZ.R} \quad (6.8)$$

This rationale is an approximation because the functional failure probabilities of categories are not linear in ECLM probability parameters. It is adopted because the estimation of parameters specifically for the categories is difficult owing to sparse data. Besides, the approximation rationale makes it easy to perform sensitivity analysis with respect to ECLM parameters and category fractions.

6.6 Random sampling method, a proposal

The development of sampling (simulation) technique-based quantification of the failure of reactivity shutdown has been proposed, combining the calculation of probability and evaluation of reactivity impact for patterns of failed rods, compare to NPSAG project proposal B.4 2007 [NPSAG-CRD-SimulProp]. Difficulties of combinatorial analysis tied to analytic approach are bypassed in sampling approach.

The use of sampling technique removes the need to express the failure criteria for control rods in simplified forms being difficult to verify unless clearly conservative. The sampling technique makes possible to consider different fuel loading configurations, core edge effects and other core conditions in an efficient way which is also expected to contribute to more realistic results in comparison to traditional approach.

Prototype experiments show that the sampling approach should be feasible, and the execution times not too long with nowadays computers.

7 Model Extensions

This chapter describes special model extensions used in practical applications in addition to the extension for localized CCFs (position correlation) of control rods and drives, handled in Section 6.4.

7.1 Component failure rate model

As underlined in basic definition section ECLM suits primarily to model failures of standby components at demand (latent failures), owing to underlying physical load-strength analogy model. ECLM applications are in fact confined within that scope, including safety and relief valves, control rods and drives, scram valves, and motor operated and check valves used as isolation valves (in highly redundant configuration).

Cases of failure modes characterized by probability of spontaneous occurrence per unit of time (failure rate) divide up in two categories:

- Failures during mission time after initiating event
- Self-revealing (monitored) failures in normal state, in the absence and/or before initiating event

Latent failures of standby components belong to in basically different category, subject to time-dependent modelling by using instantaneous unavailability as will be discussed in Section 7.2.

Considering the failure probability over whole mission time, and no repairs assumed during mission time, is similar to ordinary failure at demand. The physical analogy model also applies though the failures of redundant components (including dependent failures) can be time-separated during mission time.

It must be, however, emphasized that the mission time need to be handled as a whole, i.e. p_tot shall be associated to failure rate x mission time. It makes difference if ECLM is applied directly to hourly rate (λ_o), i.e. failure probability per hour (Δt), with $p\_tot = \lambda_o \cdot \Delta t$, and the results are thereafter multiplied by the number of hours in mission time. From the view of physical analogy model this means that failures during consecutive hours are handled as mutually independent. The difference is pronounced in low to middle order multiplicity of failure, below the order where extreme load connected failures (complete CCFs) become dominant and where naturally the total failure probability behaves linearly with respect to time. Furthermore, the statistical data for CCFs of mission time should be based on the use of CCF screening window which is compatible with mission time, i.e. not limited cover to strictly simultaneous component events which are often called as shock failures in connection to failure rate based model.

The dependence level (ECLM parameters) can differ between demand part and mission period. This can be specific to component type and length of mission period. It should also be noted that for standby components like emergency diesel generators and ECCS pumps failures at demand and during first operating hour(s) can be correlated, i.e. influenced by same failure mechanisms/CCFs. In such cases combining the first operating hour(s) with demand part is advisable, modelling only failures at later stable operating time separately.

Regarding the second type of spontaneous failures, CCFs of monitored (self-revealing) failures during normal state are usually neglected as relatively small contributors, treating those failures only as contributors to maintenance downtime of components. Using ECLM would be rather artificial. CCIs belong also to this category. They are recommended to be modelled by explicit causal models like fire risk analysis, flood analysis, seismic analysis and specific methods for other types of external hazards. It is theoretically feasible to model a CCI by triggering event with an occurrence frequency per unit of time, and conditional probability of damages to redundant components (dependent failures per demand) modelled by ECLM.

7.2 Time-dependent component unavailability model

Time-dependent extension of ECLM has been used in risk monitoring and follow-up analyses, and in the evaluation of test arrangements. The extension is based on so called $q+\lambda \cdot t$ –model used as time-dependent component model. The instantaneous unavailability of a standby component is expressed in the following form:

$$u(t) = q + (1-q) \cdot e^{-\lambda_S \cdot (t - t_{LastTest})} \cong q + \lambda_S \cdot (t - t_{LastTest}) \qquad (7.1)$$

where

q	=	Time-independent part of unavailability
λ_s	=	Standby failure rate
$t_{LastTest}$	=	Last test or demand before time point of consideration t

The approximation represents a linear model which can be used in practical applications without problems of accuracy. Failure data are needed from similar components for at least two different test intervals in order to estimate both parameters q and λ_s [TI_Opt88].

Auxiliary parameter x_{TI} is introduced to describe the relative portion of time-independent part with respect to unavailability at test:

$$x_{TI} = q/u(T) = q/(q + \lambda_s \cdot T) \qquad (7.2)$$

$$\lambda_s = (1 - x_{TI}) \cdot u(T)/T$$

where

T	=	Test interval, or generally mean time between test or demand

Conceptual basis for using ECLM is like following. A common load (demand) occurs in the end of standby time. Component condition is not known during standby. Failed component state can exist at the beginning of standby time, or be entered during standby time or realize in the end of standby time at demand. Component events need not be simultaneous. The probability of failure end states increases as the function of standby time.

The simplest way to implement time-dependent modelling for ECLM is to handle probability parameters {p_tot, p_xtr} by using the linear model, and to keep correlation coefficients {c_co, c_cx} as constant because their variability has a smaller influence. Generic insights indicate that dependence level use to increase for longer test/demand intervals of standby components. It is believed that this behaviour has still only small influence in comparison to the relevance of correlation coefficients used for the case. Compare to initial more thorough discussion of the time-dependent extension in [TVO_SRVX, Section 1]. More recently the described approach has been used for the analysis of alternative test arrangements of SRVs in Olkiluoto 1 and 2.

Ordinary estimation of time-dependence for extreme load part parameter p_xtr is practically impossible due to lack of sufficient statistics. Because the two probability parameters are usually much inter-related it makes sense to assume same linear behaviour of time-dependence in relative manner, and same relative portion of time-independent part x_{TI} first obtained for total component failure probability:

$$p\_tot(0) = x_{TI} \cdot p\_tot(T_m) \qquad (7.3)$$
$$p\_xtr(0) = x_{TI} \cdot p\_xtr(T_m)$$

Here t = 0 is associated to time point after successful test or demand, and T_m denotes mean time between test or demand during power cycle related to demand-based estimation of probability parameters.

Linear time-dependent model of probability parameters {p_tot, p_xtr} while keeping correlation coefficients {c_co, c_cx} as constant implies that SGFP entities inherit an approximately linear behavior as the function of standby time. It is hence sufficient to quantify SGFP entities at two time points, most conveniently at the beginning of standby time (after complete renewal) at t = 0 and at $t = T_m$. ECLM parameters for time point $t = T_m$ can be associated to the estimates obtained from the failure statistics, possibly partly based engineering judgment. In principle, with abundant data, the parameters of time-dependent extension could be estimated by using analogously extended Likelihood Function.

The proportionality assumption with equal x_{TI} cannot be generally applied for SGFP entities even though – with above discussed assumptions – they are crudely linear with respect to time over range $[0, 2 \cdot T_m]$. Sensitivity calculations show that the slope of relative change increases for increasing failure multiplicity. (This property actually means increasing level of dependence as the function of standby time which makes common sense, compare to the earlier discussion of this aspect.) As a corollary, SGFP entities really need to be calculated at two time points. Owing to linear transformations between different SGFP entities it is mathematically equivalent to use any of them as primary entity for time-dependent calculation.

Time-dependent and time-independent parts can be modelled also separately, forming two CCF groups corresponding to the failure modes. Furthermore, if time-independent part is considered negligible, the extension can be adapted to hold only time-dependent part.

7.3 Asymmetric CCF groups

ECLM can be extended in various ways to handle non-homogeneity (asymmetry) within a CCF group. The model load can be decomposed into parts that are specific to different subgroups. Alternatively, the component resistances can be shifted relatively to each other. These techniques can also be combined. A specific type of extensions to asymmetric case was already presented for localized CCFs of control rods and drives in Section 6.4.

Another type of extension is presented here for the situation where the CCF group constitutes of two subgroups of differences in component design, like old components and new components with some design improvements. In addition, different component age breaks some dependencies between the design subgroups. This type of extension has been applied in modelling of SRVs of Olkiluoto 1 and 2 after modernization where new SRVs were installed in addition to old SRVs.

It is assumed, that fraction z_{AB} of CCF mechanisms are common to subgroups A and B. The assumed decomposition of CCF mechanisms is described in ECLM by the following extended stress distribution:

$$f_{Stress}(x_A, x_B) = (1 - z_{AB}) \cdot f_A(x_A) \cdot f_B(x_B) + z_{AB} \cdot f_{AB}(x_A) \cdot \delta(x_B - x_A) \quad (7.4)$$

where

$\delta(\bullet)$ = Dirac's delta-function

All stress distributions for the subgroups and their combination (on the right hand side) are assumed identical, while the resistance distribution of subgroup B is shifted relative to subgroup A (fixed as base group) in order to describe the reliability difference. Differing total component failure probability is accomplished in this way. Components of subgroups have in this extension different parameter p_tot while other ECLM parameters are same. Design diversity is described by fraction z_{AB} in addition to difference in total component failure probability.

Due to the linearity of the stress-resistance expression in ECLM, the above compound stress function translates into similar decomposition of SGFP entities, giving the probability that specific number k_A, k_B of components fails in subgroups A, B, respectively (size of whole group is $n = n_A + n_B$):

$$Psg_{Sys}(k_A, k_B) = (1 - z_{AB}) \,.\, Psg_A(k_A) \,.\, Psg_B(k_B) + z_{AB} \,.\, Psg_{AB}(k_A, k_B) \qquad (7.5)$$

Index 'Sys' refers to system (whole group) failure. Psg_{Sys} and Psg_{AB} are two-dimensional arrays. Other SGFP entities have corresponding definition in the asymmetric case of two design groups:

$$Peg(k_A, k_B|n_A, n_B) = P\{\text{ Specific } k_A, k_B \text{ of the components fail in subgroups A, B, respectively, while the other components survive}\} \qquad (7.6)$$

$$Pes(k_A, k_B|n_A, n_B) = P\{\text{ Some } k_A, k_B \text{ but no more of the components fail in subgroups A, B, respectively }\}$$

$$Pts(k_A, k_B|n_A, n_B) = P\{\text{ Some } k_A, k_B \text{ or more of the components fail in subgroups A, B, respectively}$$

These can be derived from Psg-entities by using transformations that are straightforward extensions of the transformations in symmetric base case (one-dimensional SGFP arrays). The dimensions are treated independently in combinatorial analysis of two-dimensional SGFP arrays. Compare to similar extension in [TC_PASDG].

It is equivalent with the described approach to model separately the failure and CCF mechanisms affecting the whole group homogeneously, and separately those affecting exclusively design subgroups. CCF groups are formed for all components and for each design subgroup, and the groups are assumed mutually independent. The results can be combined in PRA fault trees, or off-line in the presented array form.

8 Comparison with Alternative Models

Insights of an earlier model review and comparison will be shortly summarized here, more details are presented in Ref.[SKI TR-91:6, HR_CCFRe].

The mostly used CCF models, Multiple Greek Letter Method (MGLM) and Alpha Factor Method (AFM), were already discussed in Section 4.4 with regard to advantages and drawbacks in handling demand subgroups and constructing interface to PRA models. A remarkable drawback of AFM and MGLM is the addition of a new model parameter for each degree of redundancy. For CCF groups of more than 10 components there becomes unnecessarily many parameters when compared to in which detail actual data exists about high multiplicity failures and dependencies. Leaving away some of the intermediate parameters results in a model which is difficult to track with respect to whether the event combinatorics is handled properly (experiments in this direction are discussed in [HR_CCFRe]). Handling of a large number of Alpha Factors or MGLM parameters would also be very laborious because they are not subgroup-invariant.

This contrasts to ECLM, which includes four parameters, fulfilling the desired property of subgroup invariance.

The more viable alternatives for a CCF model in highly redundant cases can be found among ordinary parametric models. This concerns specially the Binomial Failure Rate Model and its variant, Multi-Binomial Model (MBM), adapted to demand failure probability model [HR_CCFRe]. As MBM is defined in terms of SGFPs, it fulfils the desired property of subgroup invariance. However, more than one nonlethal shock term seems to be needed in order to more generally describe dependence patterns in CCF groups of sizes above n=10, which makes at least six model parameters.

In the connection to the recent review of ECLM, so called Beta CCF Method was proposed. This model contains only two parameters. It is thus a trivial conclusion that this proposal cannot be adequate for modelling of CCFs in highly redundant systems, see [NKS 90-2003].

9 Discussion of Experiences

The principal advantage of ECLM is that the stress-resistance expression is a comprehensive description of the dependence mechanisms for any group of identical components – limited only by the simplifications in the stress and resistance distributions used for a practical implementation of the concept. In principle, all shared caused mechanisms can be covered by the common stress variable and its distribution. But also indirect CCF mechanisms such as systematic degradation (e.g. due to variations in operating environment or maintenance quality) can be effectively described by the stress distribution. On the other hand, resistance distribution provides room for variation from component-to-component. Therefore, ECLM has expressive power of more general type than the approaches based on CCF basic event model.

ECLM is a particular specialization of the stress-resistance expression as described in this report. It contains four model parameters but proves nevertheless rather flexible to practically adequate description of dependences in internally homogeneous groups up to the extent as there are available statistics of high order failures.

ECLM applies well to the cases where the time variable can be reduced to the consideration of failures at a random time point, at which a demand is imposed on standby components, and where the component operability/failure conditions can be assumed equal and symmetric within the group. The model can also be applied to failure over mission time.

Further extensions of the stress/resistance distribution model can be made when the standby components are tested with different intervals or non-symmetric time scheme. Also, there may exist design diversity which breaks the internal symmetry. ECLM adapts well to extensions for non-homogeneous groups, for example, in order to describe asymmetric diversity against CCF mechanisms. CCF mechanisms may also be localized in such a way that adjacent components are more likely affected than the component separated by distance such as in the case of control rods and drives.

The main limitation is generally sparse statistical data of CCFs which problem is shared by all parametric CCF models. The problem is pronounced with the model extensions because the additional model features should be verified by operational experience. For the time being, much is based on engineering judgment. The implementation of Bayesian estimation methods can help to utilize available statistics and related knowledge more efficiently. Further development is especially desired on techniques to combine information from different sources taking into account the evidence about the impact of design differences and CCF defence differences.

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Acronyms

AFM	Alpha Factor Method
BFRM	Binomial Failure Rate Model
BWR	Boiling Water Reactor
CCBE	Common Cause Basic Event
CCCG	Common Cause Component Group
CCF	Common Cause Failure
CLM	Common Load Model
DC	Direct Current
ECLM	Extended Common Load Model
EPV	Electromagnetic Pilot Valve
ICDE	International CCF Data Exchange
MBM	Multi Binomial Model
MCS	Minimal Cut Set
MGLM	Multiple Greek Letter Method
NAFCS	Nordisk Arbetsgrupp för CCF studier (Nordic Workgroup for CCF Analyses)
NPSAG	Nordic PSA Group
PRA	Probabilistic Risk Assessment
PSA	Probabilistic Safety Assessment
PWR	Pressurised Water Reactor
RBMK	Russian acronym for Channelized Large Power Reactor
SGFP	Subgroup Failure Probability
SKI	Swedish Nuclear Power Inspectorate
SRV	Safety and Relief Valve
TVO	Teollisuuden Voima Oyj

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Appendix 1

Mathematical Details of the Parametrization

This appendix explains the details of the parametrization applied in the Extended Common Load Model (ECLM).

Contents

A1-1 Stress-Resistance Distributions

The common load is constructed of two distribution parts, compare to Fig.A1-1 (the diagram uses rescaled variable y, which is defined in Section A1-2):

$$f_S(x) = w_b \cdot f_{Sb}(x) + w_x \cdot f_{Sx}(x) \qquad \text{(A1-1.1)}$$

where

$f_{Sb}(x)$ = Base load part, density function

$$= \frac{1}{\sigma_{Sb}} . Z\left(\frac{x-\mu_{Sb}}{\sigma_{Sb}}\right)$$

$f_{Sx}(x)$ = Extreme load part, density function

$$= \frac{1}{\sigma_{Sx}} . Z\left(\frac{x-\mu_{Sx}}{\sigma_{Sx}}\right)$$

Z(•) denotes (0,1)-normal distribution density function. The stress distributions are basically determined by means μ_{Sb},μ_{Sx} and standard deviations σ_{Sb},σ_{Sx}: Weight factors sum up to 1:

$$w_b + w_x = 1 \qquad \text{(A1-1.2)}$$

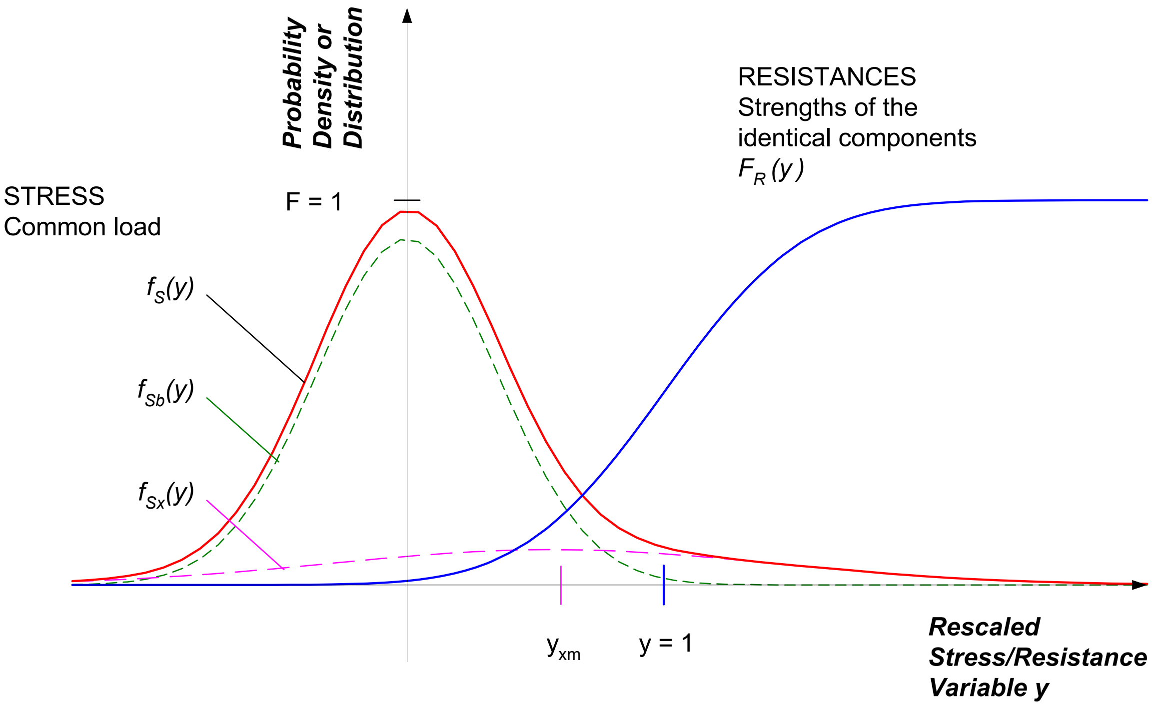

Figure A1-1 Schematic presentation of the stress-resistance distributions.

Correspondingly, the total single failure probability is divided up into base load and extreme load parts:

$$\begin{aligned} psg(1) &= p\_tot \\ &= p\_bas + p\_xtr \\ &= w_b \,.\, P_{1b} + w_x \,.\, P_{1x} \end{aligned} \qquad \text{(A1-1.3)}$$

where p_tot and p_xtr are chosen as model parameters. The auxiliary variables P_{1b} and P_{1x} denote the respective probability integral:

$$P_{1b} = \int_{x=-\infty}^{\infty} dx.f_{Sb}(x).F_R(x)$$

$$P_{1x} = \int_{x=-\infty}^{\infty} dx.f_{Sx}(x).F_R(x) \qquad \text{(A1-1.4)}$$

The resistance distribution is determined by mean μ_R and standard deviation σ_R:

$$F_R(x) = F\left(\frac{x-\mu_R}{\sigma_R}\right)$$

F(•) denotes standard (0,1) normal distribution function. For practical convenience its complementary function 1 – F(•) is preferred in the numeric analysis, and will be denoted as:

$$P_{Norm}(\bullet) = 1 - F(\bullet) \qquad \text{(A1-1.5)}$$

The reverse function of $P_{Norm}(\bullet)$will be denoted as $x_{Norm}(\bullet)$.

A1-2 Rescaling, Standard Deviations

The stress/resistance variable x will be rescaled with respect to the interval $[\mu_{Sb},\mu_R]$ through the following substitution:

$$y = \frac{x-\mu_{Sb}}{\mu_R-\mu_{Sb}} \Rightarrow \frac{dy}{d_{Sb}} = \frac{dx}{\sigma_{Sb}} \qquad \text{(A1-2.1)}$$

with corresponding renormalization of standard deviations:

$$d_{Sb} = \frac{\sigma_{Sb}}{\mu_R - \mu_{Sb}}$$

$$d_{Sx} = \frac{\sigma_{Sx}}{\mu_R - \mu_{Sb}}$$

$$d_R = \frac{\sigma_R}{\mu_R - \mu_{Sb}}$$

The rescaling converts the argument in the base load part into y/d_{Sb}, and in the extreme load part in the following way:

$$\begin{aligned}\frac{x-\mu_{Sx}}{\sigma_{Sx}} &= \frac{x-\mu_{Sb}-(\mu_{Sx}-\mu_{Sb})}{\sigma_{Sx}} \\ &= \frac{y.(\mu_R-\mu_{Sb})-(\mu_{Sx}-\mu_{Sb})}{\sigma_{Sx}} \\ &= y.\frac{1}{d_{Sx}} - y_{xm}.\frac{1}{d_{Sx}} \\ &= \frac{y-y_{xm}}{d_{Sx}}\end{aligned} \qquad \text{(A1-2.2)}$$

Where

y_{xm} = Rescaled mean point of the extreme load part

$$= \frac{\mu_{Sx}-\mu_{Sb}}{\mu_R-\mu_{Sb}} \qquad \text{(A1-2.3)}$$

Similarly, the argument in the resistance distribution is converted in the following way:

$$\begin{aligned}\frac{x-\mu_R}{\sigma_R} &= \frac{x-\mu_{Sb}-(\mu_R-\mu_{Sb})}{\sigma_R} \\ &= \frac{y.(\mu_R-\mu_{Sb})-(\mu_R-\mu_{Sb})}{\sigma_R} \\ &= \frac{y-1}{d_R}\end{aligned} \qquad \text{(A1-2.4)}$$

A1-3 Integral Expression

The stress/resistance probability integral is converted by the rescaling into the following form:

$$psg(k) = \int_{x=-\infty}^{\infty} dx.f_S(x).[F_R(x)]^k$$

$$= \int_{x=-\infty}^{\infty} dx.\left[\frac{w_b}{\sigma_{Sb}}.Z\left(\frac{x-\mu_{Sb}}{\sigma_{Sb}}\right)+\frac{w_x}{\sigma_{Sx}}.Z\left(\frac{x-\mu_{Sx}}{\sigma_{Sx}}\right)\right].\left[F\left(\frac{x-\mu_R}{\sigma_R}\right)\right]^k \quad \text{(A1-3.1)}$$

$$= \int_{x=-\infty}^{\infty} dy.\left[\frac{w_b}{d_{Sb}}.Z\left(\frac{y}{d_{Sb}}\right)+\frac{w_x}{d_{Sx}}.Z\left(\frac{y-y_{xm}}{d_{Sx}}\right)\right].\left[F\left(\frac{y-1}{d_R}\right)\right]^k$$

A1-4 Correlation Coefficients

The following correlation coefficients are defined in analogy to the original CLM parametrization, and these are set as model parameters:

$$c\_co = \frac{d_{Sb}^2}{d_{Sb}^2 + d_R^2} \quad \text{(A1-4.1)}$$

$$c\_cx = \frac{d_{Sx}^2}{d_{Sx}^2 + d_R^2}$$

Alternative, the ratios d_R/d_{Sb}, d_R/d_{Sx} could be used as "dependence parameters". However, c_co, c_cx are preferred because their value range makes sense together with fact that value 0 means independence and 1 total dependence, compare to the discussion of the parameter choice in the original introduction to CLM.

A1-5 Total Single Failure Probability

The connection to the decomposition of the total single failure probability in Eq.(A1-1.3) means that

$$\begin{aligned}
p_bas &= p_tot - p_xtr \\
&= w_b . \int_{y=-\infty}^{\infty} \frac{dy}{d_{Sb}} . Z\left(\frac{y}{d_{Sb}}\right) . F\left(\frac{y-1}{d_R}\right) \\
&= w_b . P_{Norm}\left(\frac{1}{\sqrt{d_{Sb}^2 + d_R^2}}\right) \qquad \text{(A1-5.1)} \\
&= w_b . P_{Norm}(xn_bas) \\
&= w_b . P_{1b}
\end{aligned}$$

where the following auxiliary variable is defined

$$xn\_bas = \frac{1}{\sqrt{d_{Sb}^2 + d_R^2}}$$

Similarly

$$\begin{aligned}
p_xtr &= w_x . \int_{y=-\infty}^{\infty} \frac{dy}{d_{Sx}} . Z\left(\frac{y-y_{xm}}{d_{Sx}}\right) . F\left(\frac{y-1}{d_R}\right) \\
&= w_x . P_{Norm}\left(\frac{1-y_{xm}}{\sqrt{d_{Sx}^2 + d_R^2}}\right) \qquad \text{(A1-5.2)} \\
&= w_x . P_{Norm}(xn_xtr) \\
&= w_x . P_{1x}
\end{aligned}$$

where the following auxiliary variable is defined

$$xn\_xtr = \frac{1-y_{xm}}{\sqrt{d_{Sx}^2 + d_R^2}}$$

A1-6 Reverse Solution for Distribution Parameters as the Function of Model Parameters

The parameters p_tot, p_xtr, c_co, c_cx are chosen as macro parameters of ECLM. For the numeric integration of the stress/resistance equation, the distribution parameters are obtained by the following reverse solution.

In the first step of the reverse solution the following relationship is obtained from Eq.(A1-5.2) for variable P_{1x}:

$$P_{1x} = P_{Norm}(xn\_xtr)$$

and then with the reverse solution, using the normal distribution point $x_{Norm}(\bullet)$ as the function of cumulative distribution value:

$$xn\_xtr = x_{Norm}(P_{1x}) = \frac{1-y_{xm}}{\sqrt{d_{Sx}^2 + d_R^2}} = \frac{1-y_{xm}}{d_R}.\sqrt{1-c\_cx} \qquad \text{(A1-6.1)}$$

In the last stage, the relationship given in Eq.(A1-4.1.b) was used.

A1-6.1 Reduction of free variables

The total number of parameters is basically 8: two for each three normal distributions and two auxiliary weights w_b and w_x. But these weights are bound by the following normalization equation:

$$w_b + w_x = 1 \qquad \text{(A1-6.2a)}$$

which is equivalent to

$$\int_{x=-\infty}^{\infty} dx.f_S(x) = \int_{x=-\infty}^{\infty} dx.\left(w_b.f_b(x) + w_x.f_x(x)\right) = 1 \qquad \text{(A1-6.2b)}$$

as both $f_b(x)$ and $f_x(x)$ are normalized to one. Thus there are basically 7 free parameters. One parameter is reduced by the fact that the linear translation does not matter: the mean of base load distribution is fixed to origin (when using the rescaled stress/resistance variable y):

$y_{bm} = 0$ (A1-6.3a)

Another choice to fix the scale is the placement of the mean of resistance distribution:

$y_{Rm} = 1$ (A1-6.3b)

This choice does not yet reduce the generality of implementation because the relative spread of the three distributions and the placement of extreme load part as well as the ratio between extreme and base load part effectively determine the outcome probabilities. So there are 5 effective free variables at this point.

The placement of the extreme load distribution proved out to be difficult to express by a meaningful model parameter. After a series of trials with many different parametrization alternatives, it came up that the overall relationship between the parameters is greatly simplified by the following anchoring:

$y_{xm} = 1 - d_R$ (A1-6.3c)

This choice reduces the coupling between model parameter pairs {p_tot, c_co} and {p_xtr, c_cx}, which showed up practically convenient.

The mean of f_{Sx} is thus anchored at a distance of one standard deviation of the resistance distribution, to the left from the mean of the resistance distribution (usually between means of f_{Sb} and f_R because in practical cases $d_R < 1$). The chosen anchoring implies that, see Eq.(A1-6.1):

$$xn\_xtr = \sqrt{1 - c\_cx} \quad \text{(A1-6.4.a)}$$

and consequently, P_{1x} becomes dependent of the extreme load part correlation only:

$$P_{1x} = P_{Norm}\left(\sqrt{1 - c\_cx}\right) \quad \text{(A1-6.4.b)}$$

Fixing the distribution placements in the above described way means that there are left 4 degrees of freedom, which are represented by the four defined model parameters. At his point the following four distribution variables remain still to be expressed in terms of the model parameters:

$\{ w_X, d_{Sb}, d_{Sx}, d_R \}$ (A1-6.5)

The relative placement of base load mean to the left from resistance mean implies that $P_{1b} \leq 0.5$, similarly p_tot is limited below ~0.5. There are correspondingly theoretical limits of other parameters close to the extreme values beyond practically meaningful ranges, not discussed here.

A1-6.2 Expressions for the remaining distribution variables

The open distribution variables in set (A1-6.5) will then be expressed in terms of macro parameters. Firstly, by using Eqs.(A1-1.2-3):

$$w_x = \frac{p\_xtr}{P_{1x}} \quad \text{(A1-6.6.a)}$$

$$w_b = 1 - w_x \quad \text{(A1-6.6.b)}$$

$$P_{1b} = \frac{p\_bas}{w_b} = \frac{p\_tot - p\_xtr}{w_b} \quad \text{(A1-6.6.c)}$$

Using Eq.(A1-5.1) and inserting c_co from Eq.(4.1.a):

$$xn\_bas = x_{Norm}(P_{1b}) = \frac{1}{\sqrt{d_{Sb}^2 + d_R^2}} \quad \text{(A1-6.7.a)}$$

$$= \frac{1}{d_{Sb}} . \sqrt{c\_co} \quad \text{(A1-6.7.b)}$$

$$= \frac{1}{d_R} . \sqrt{1 - c\_co} \quad \text{(A1-6.7.c)}$$

These lead to the following solutions:

$$d_{Sb} = \frac{1}{xn\_bas} . \sqrt{c\_co} \quad \text{(A1-6.8.a)}$$

$$d_R = \frac{1}{xn\_bas} . \sqrt{1 - c\_co} \quad \text{(A1-6.8.b)}$$

Finally, rearranging Eq.(4.1), the following expression is derived:

$$d_{Sx} = d_R . \sqrt{\frac{c\_cx}{1 - c\_cx}} \quad \text{(A1-6.9)}$$

A1-7 Summary of Variable Expressions

In summary, the required set of distribution variables for the rescaled stress/resistance expression, Eq.(A1-3.1), can be derived through the following steps:

Calculation step			Compare to Eq.
w_x	$= \frac{p\_xtr}{P_{1x}}$	with $P_{1x} = P_{Norm}\left(\sqrt{1-c\_cx}\right)$	(A1-6.4.b)
wb	= 1 - wx		(A1-1.2)
dSb	$= \frac{1}{xn\_bas}\cdot\sqrt{c\_co}$	with xn_bas = xNorm(P1b)	(A1-6.8.a)
		and $P_{1b} = \frac{p\_tot - p\_xtr}{w_b}$	(A1-6.6.c)
dR	$= \frac{1}{xn\_bas}\cdot\sqrt{1-c\_co}$		(A1-6.8.b)
dSx	$= d_R\cdot\sqrt{\frac{c\_cx}{1-c\_cx}}$		(A1-6.9)
yxm	= 1 - dR		(A1-6.3c)

In HiDep program, the integration is performed numerically by using the standard approximations for the normal probability distribution $F(\bullet) = 1-P_{Norm}(\bullet)$ and its reverse function $x_{Norm}(\bullet)$, as presented in [A&S, Sections 26.2.17 and 26.2.23].

A1-8 Sensitivity to Model Parameters

As already discussed the used definition of model parameters has the advantage that parameter pairs {p_tot, c_co} and {p_xtr, c_cx} have intuitively anticipated influence on outcome probabilities and dependence profile, and weak mutual coupling is achieved. Certain relationships exist, however. They are good to know because else the behaviour of model outcome may look unexpected, e.g. in sensitivity analyses, even though the peculiarities are quantitatively small. This section continues the general discussion in Report Section 3.2.

The two probability parameters p_tot, p_xtr are straightly present in the breakdown of the total component failure probability, and in that way related:

$$
\begin{aligned}
p\_tot &= p\_bas + p\_xtr \\
&= Psg(1) \\
&= (1-w_x)\cdot P_{1b} + w_x\cdot P_{1x} \qquad \text{(A1-8.1)}
\end{aligned}
$$

The choice of p_tot as one model parameter is a practical must. In sensitivity analyses for parameters it is meaningful to allow p_tot float when varying parameters p_xtr, c_co and c_cx, and retain probability level of base load part by keeping p_bas constant, or preferably P_{1b} as constant in order to isolate the coupling of weight factors as will be discussed later in more detail.

Weight factor w_x is relatively simply connected to extreme load part parameters with linear connection to p_xtr and rather weak negative coupling to c_cx within the typical range of parameters:

$$w_x = \frac{p\_xtr}{P_{1x}} = \frac{p\_xtr}{P_{Norm}(\sqrt{1-c\_cx})} \qquad \text{(A1-8.2)}$$

Multiple failure probability can be presented in the following form in order to show parameter relationships:

$$Psg(m) = (1-w_x)\cdot P_{mb}(d_R, d_{Sb}) + w_x\cdot P_{mx}(d_R, d_{Sx}) \qquad \text{(A1-8.3)}$$

Here the shorthand notations P_{mb} and P_{mx} are used for the integration terms. The derivation of the above breakdowns uses several equations handled in the previous sections. The base load part is indeed primarily determined by parameters {p_tot, c_co} as can be verified from Eqs.(A1-6.6 and 6.8) and taking into account that p_xtr is small compared to p_tot, and $w_b = 1-w_x$ close to one in practical cases.

Common normalization of correlation coefficients by using d_R implies that correlation coefficient c_co influences all standard deviations while c_cx influences only d_{Sx}, compare to summarized expressions in Section A1-7. As a consequence, model parameters p_xtr and c_cx influence primarily extreme load part. They affect base load part via normalization $w_b = 1-w_x$ but only weakly because w_x is in practical cases small or very small. Parameter p_xtr influences linearly as a relative factor via w_x to extreme load part probabilities $Psg_{xtr}(m)$, being thus the simplest model parameter to understand with respect to relationships. Parameter c_cx influences via w_x probability level (rather weak negative coupling) and via d_{Sx} to dependence profile of extreme load part (positive coupling to $Psg_{xtr}(m)$ for m>1). The influence of parameter pair {p_xtr, c_cx} concentrate thus on extreme load part, being much simpler than influence of parameter pair {p_tot, c_co}.

It is not of interest in practice to perform sensitivity analysis with respect to p_tot (or p_bas), keeping other model parameters as constant. From a theoretical point of view it is worth to notice that p_tot (or p_bas) affects base load part directly via P_{1b} but also all standard deviations. This has remarkable influence on the low order probabilities. The influence to extreme load part is practically small. (Couplings are globally positive.)

It is important to notice that strictly taken it is not possible to keep both p_bas = (1- w_x)·P_{1b} and P_{1b} constant when varying parameter pair {p_xtr, c_cx} except if w_x is kept constant, and parameters p_xtr and c_cx are varied as coupled via Eq.(A1-8.2). It is preferred to retain the probability level of base load part by keeping P_{1b} constant in individual variation of p_xtr or c_cx, because then standard deviations are less affected.

In summary following specific remarks can be presented for parameter variations.

p_xtr: Sensitivity to model parameter p_xtr is simple, especially if probability level of base load part is retained by keeping P_{1b} constant. In combination with c_co and c_cx constant this implies that distribution variables are unchanged, and Psg_{xtr}(m) varies linearly with respect to p_xtr. The probability level of base load part changes very little in opposite direction because of small influence via weight factors.

c_cx: Parameter c_cx affects first of all the variance of extreme load (dS_x) which is seen in Psg_{xtr}(m) in growing degree at increasing multiplicity, i.e. as slope change (positive coupling). It influences also the probability level of extreme load part via weight factor w_x (negative coupling) but this influence can be masked by the stronger influence via distribution variance. The probability level of base load part changes even for varied c_cx, also very little in opposite direction because of small influence via weight factors.

c_co: Parameter c_co affects all standard deviations. The main impact is, however, the slope change in the dependence profile of base load part (positive coupling). Other influences are relatively small, usually even very small.

The behavior of Psg entities in parameter variations is intuitively simple to anticipate for base load part, extreme load part and in total. The changes for other SGFP entities are complicated because of interference from combinatorial aspects. The behaviour of Pts(1|n) can look in the first instance anomalous in certain parameter combinations as it can decrease for increased c_co and vice versa while at other orders Pts(m|n) is positively coupled to c_co. This aspect can be illustrated in simplest way in a system of two components where (P_m denotes Psg(m))

$$Pts(1|2) = 2 \cdot P_1 - P_2, \text{ and } Pts(2|2) = P_2 \qquad \text{(A1-8.4)}$$

If dependence among the components is increased ($\Delta P_2 > 0$) so that $\Delta P_1 < ½ \cdot \Delta P_2$ then Pts(1|2) will decrease “surprisingly”. Similarly, the increase (decrease) of parameter c_cx can in certain cases cause decrease (increase) of Pts(m|n) at low to intermediate order (negative coupling) while else the coupling is positive.

It has to be mentioned that earlier the limited numerical accuracy of ECLM integration procedure disturbed the output of sensitivity calculations. At this point the numerical approximations used for normal probability functions can cause some small but qualitatively disturbing discrepancies in parameter variations. Especially, the inverse $X_{Norm}(\bullet)$ is not very accurate, compare to last paragraph of Section A1-7. It would be good to replace that with a better approximation, preferably such one that $X_{Norm}(P_{Norm}(x))$ is sufficiently close to x. The problems with numerical accuracy can become pronounced at extreme values of model parameters. The numerical accuracy can also affect derived maximum of likelihood function in cases with flat surface top; the maximum is then also much uncertain from statistical point of view.

All in all, it is advisable to track changes in distribution variables during parameter variations in order to have control what is happening in the sensitivity calculations.

A1-9 References

CLM_77	Mankamo, T., Common Load Model, a tool for Common Cause Failure analysis. Technical Research Centre of Finland, Electrical Engineering Laboratory, Report 31, December 1977.
HiDep	HiDep, CCF Analysis Toolbox, Version 2.5. Avaplan Oy, 2006.
A&S	Abramowiz&Segun, Handbook of Mathematical Functions. Dover 1971.

Appendix 2

Localized CCF Mechanisms of Control Rods, Application of Common Load Model, Mathematical Details

This appendix presents the mathematical details of model extension to handle localized CCFs of control rods. 'Report' is used here to refer to the contents of the report body.

Contents

A2-1 Modelling Assumptions

A2-1.1 Scope

The extension for modelling localized CCF mechanisms of control rods is generally described in Report Section 6.4, including the reasoning behind key assumptions. This appendix focuses on the mathematical details, including parametrization of the extension. This presentation updates earlier description [CLM-LocZ-1995]. This presentation serves also theoretical basis for the practical calculation tool in HiDep Toolbox. Therefore, variable notation resembles the convention of programming languages.

A2-1.2 Basic concepts, shell layout

Localized CCF mechanisms are basically assumed to first affect one control rod denoted as Rod ∅. The failure can next escalate to adjacent rods which constitute inner shell in the two-dimensional map of control rods, in horizontal cross-section of core, see Figure A.2-1. The next adjacent rods of outer shell can fail with lower conditional failure probability. In the model extension this is accomplished by shifting resistance distribution to right but else retaining the model setup. This means one additional model parameter.

Retaining the standard deviations is equivalent to assuming that correlation coefficients are same for outer and inner shell components. The model extension is built on dividing the CCF group into two subgroups. The first affected component (Rod ∅) is handled belonging to the subgroup of inner shell components. The other subgroup is constituted of outer shell components.

Failure criterion is defined by using Minimal Critical Shape, and requiring that its placements have to contain Rod ∅. Failures beyond the two shells are neglected, i.e. null conditional failure probability is assumed for eventual placements of Minimal Critical Shape which contain distant rod positions (and Rod ∅).

Localization (position correlation) is divided up in two types, radial and band correlated types, each with specific shell layout. Compare to Report Figure 6-2 which shows the configurations used in recent applications. Indexing of relative rod positions is specific to shell layout configuration and correlation type similarly as the set of different placements of Minimal Critical Shape.

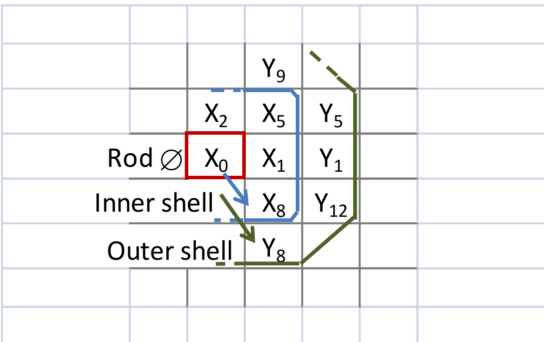

Figure A2-1 Shell model of localized CCFs.

A2-2 ECLM Probability Expression and Parameters

A2-2.1 Probability expression

The probability that specific number of inner shell components (k_{is}) and outer shell components (k_{os}) fail is obtained by the following integration:

$$Psg_{LocZ}(k_{is}, k_{os}) = \int_{y=-\infty}^{\infty} dy \cdot \left[\frac{w_b}{d_{Sb}} \cdot Z\left(\frac{y}{d_{Sb}}\right) + \frac{w_x}{d_{Sx}} \cdot Z\left(\frac{y - y_{xm}}{d_{Sx}}\right)\right] \cdot F\left(\frac{y-1}{d_R}\right)^{1+k_{is}} \cdot F\left(\frac{y - y_{om}}{d_R}\right)^{k_{os}} \quad \text{(A2-1)}$$

Compare to the analogous expression using rescaled variables for the base implementation in Appendix 1, Eq.(A1-3.1). Here y_{om} denotes the mean of the shifted resistance distribution for outer shell components, see Figure A2-2.

The failure of Rod ∅ is represented by indexes (0, 0). It is mathematically handled as a member of inner shell CCF subgroup. Psg_{LocZ}(-1, 0) presents no failure event (equal to one). Elements (0, k_{os}) for $k_{os} > 0$ represent theoretical failure combinations of Rod ∅ and some outer shell components and no inner shell components. These are virtual cases because contributing failure cases (relevant placements of Minimal Critical Shape) all contain one or more inner shell components. This is well illustrated in the example in Section 3.

The probability calculations can be arranged by using Psg entity, thus the other SGFP entities are not needed in the quantification of localized CCF mechanisms as will be explained in Section 3. It should be noticed that in eventual transformations the range of k_{is} is [-1, n_{is}] while k_{os} runs over normal range [0, n_{os}].

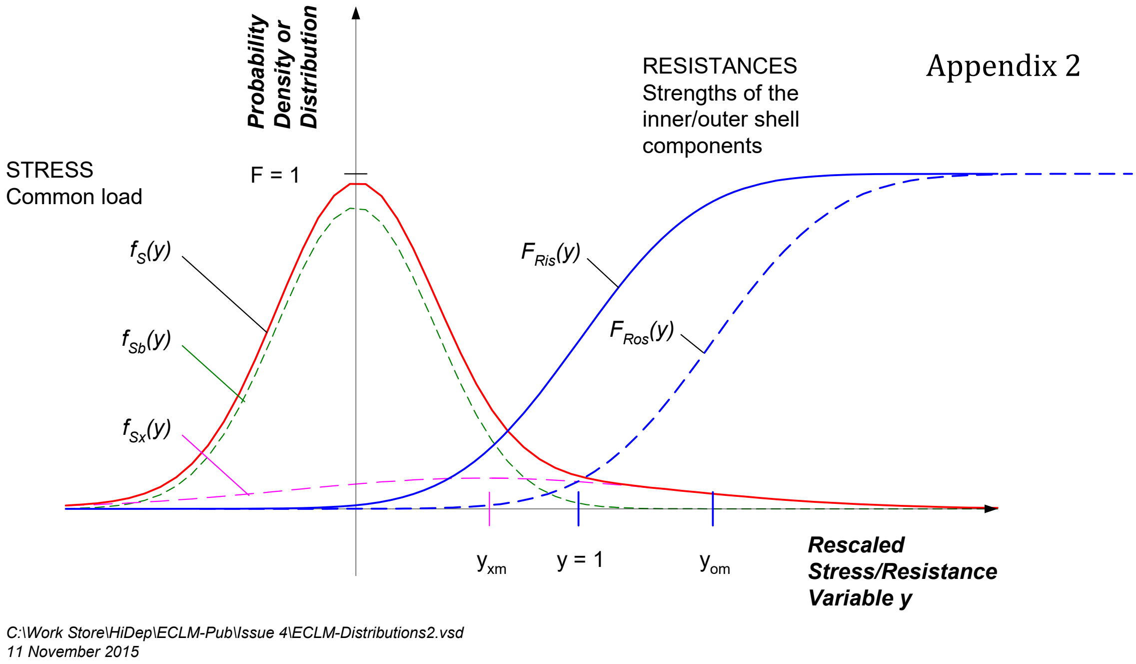

Figure A2-2 Stress and resistance distributions of the model extension.

A2-2.2 Inner shell dependence parameters

The dependence among inner shell group is modelled by parameter pair {p_tot, c_co} for base load part and by parameter pair {p_xti, c_cx} for extreme load part similarly as in the base implementation of eCLM. For improved clarity the probability parameter of extreme load part is denoted here as p_xti for inner shell components. This notation ensures consistency with using p_xto for the probability parameter of extreme load part of outer shell components, a new model parameter. The distribution variables – for common stress and for resistance distribution of inner shell components –are connected in the same way to the model parameters of inner shell group as in the base implementation, the connecting equations are presented in Appendix 1.

It is peculiar to the extension (actually to the used indexing) that total component failure probability in inner shell group is represented by $Psg_{LocZ}(0, 0)$ = p_tot, i.e. it is associated to the failure of Rod ∅, not taking into account the status of other components. It has similar breakdown to base load and extreme load parts as in the base implementation which was discussed in Appendix 1. It should be noticed that element $Psg_{LocZ}(1, 0)$ represent the failure of Rod ∅ and one specific inner shell component. The issue of total component failure probability will be discussed further in Section A2-2.4.

A2-2.3 Outer shell dependence parameters

Component resistance distribution is shifted to right for outer shell components in comparison to inner shell components, compare to Figure A2-2. The mean is placed at:

$$y_{om} = 1 + (u_{out} - 1) \cdot d_R \quad \text{(A2-2)}$$

Auxiliary variable u_{out} has value ≥ 1, similarly y_{om} per definition. The first '1' on the right hand side represents the mean of resistance distribution for inner shell components. The shift of resistance distribution implies that the probability level of outer shell components is lower than for inner shell components.

Total failure probability of outer shell components is represented by Psg_{LocZ}(-1, 1), and denoted here by p_toto. Its breakdown is following, compare to Appendix 1, Eqs.(A1-5.1 and 2):

$$p\_toto = w_b \cdot P_{Norm}\left(\frac{y_{om}}{\sqrt{d_{Sb}^2+d_R^2}}\right) + w_x \cdot P_{Norm}\left(\frac{y_{om}-y_{xm}}{\sqrt{d_{Sx}^2+d_R^2}}\right) \qquad (A2\text{-}3)$$

Load fractions w_b and w_x are same as for inner shell components related to the assumption that stress distribution is common. The latter term of the breakdown is defined as the probability parameter for extreme load part of outer shell components p_xto, a new model parameter. It can be further developed in the following way:

$$p\_xto = w_x \cdot P_{Norm}\left(\frac{1+(u_{out}-1)\cdot d_R-(1-d_R)}{\sqrt{d_{Sx}^2+d_R^2}}\right) \qquad (A2\text{-}4)$$

$$= w_x \cdot P_{Norm}\left(\frac{u_{out} \cdot d_R}{\sqrt{d_{Sx}^2 + d_R^2}}\right)$$

$$= w_x \cdot P_{Norm}(u_{out} \cdot xn\_xtr)$$

Auxiliary variable xn_xtr is defined with the following connection to probability parameter for extreme load part of inner shell components, here denoted as p_xti, compare to Eqs.(A1-5.2 and 6.3.c):

$$xn\_xtr = \sqrt{1-c\_cx} = \sqrt{\frac{d_R^2}{d_{Sx}^2+d_R^2}} \qquad (A2\text{-}5)$$

$$p\_xti = w_x \cdot P_{Norm}(xn\_xtr)$$

Reduction in the probability parameter for extreme load part becomes thus:

$$\frac{p\_xto}{p\_xti} = \frac{P_{Norm}(u\_out \cdot xn\_xtr)}{P_{Norm}(xn\_xtr)} \qquad (A2\text{-}6)$$

Auxiliary variable u_{out} can be solved using the last equation sets into the following way:

$$u_{out} = \frac{x_{Norm}\left(\frac{p\_xto}{w_x}\right)}{xn\_xtr} = \frac{x_{Norm}\left(\frac{p\_xto}{w_x}\right)}{x_{Norm}\left(\frac{p\_xti}{w_x}\right)} \qquad \text{(A2-7)}$$

Mean y_{mo} can then be expressed in terms of model parameters using its definition, Eq.(A2-2). Other distribution variables can be obtained from model parameters in the same way as in the base implementation. Their derivation is summarized in Appendix 1, Section 7.

It is of interest to develop also the first term of the breakdown in Eq.(A2-3), i.e. base load part probability of outer shell components. It can be expressed in the following way using auxiliary variable xn_bas defined in Appendix 1, Eq.(A1-5.1):

$$p\_baso = w_b \cdot P_{Norm}(y_{om} \cdot xn\_bas) \qquad \textit{(A2-8)}$$

Altogether following breakdown is thus obtained

$$\begin{aligned} p\_toto &= p\_baso + p\_xto \\ &= w_b \cdot P_{Norm}(y_{om} \cdot xn\_bas) + w_x \cdot P_{Norm}(u\_out \cdot xn\_xtr) \end{aligned} \qquad \text{(A2-9)}$$

A2-2.4 Further remarks about the model extension

As described in Report Section 6.4 model parameter p_tot of the extension to localized CCFs is first associated to the estimated probability of all types of failures anywhere in the core and the results are at the end weighted (normalized) by the fraction of localized CCFs. The association uses approximation where parameter p_tot is set to n·<p_sgl>, the product of the number of control rods n and estimated total (single) component failure probability of all failure mechanisms denoted here as <p_sgl>. Strictly taken, the probability that one or more component fails Pts(1|n) should be used. The relationships are schematically following:

p_tot ≙ *n·<p_sgl> >≈ Pts(1|n)* *(A2-10)*

The approximation n·<p_sgl> is used because it can be set before calculations while Pts(1|n) is dependent on all model parameters. The use of latter one would necessitate a cumbersome iteration. The approximation has only small influence by side of overall uncertainties of CCF modeling. It has to be also emphasized that the other model parameters are primary for the probability of multiple failures.

When developing the described extension to localized CCFs a more sophisticated alternative was considered. The variance of resistance was increased for outer shell components in addition to shifting the mean of resistance distribution to right. This makes sense in the light of generic insights from CCF mechanisms. But the alternative would require two new model parameters: specific probability parameter and specific correlation coefficient for extreme load part of outer shell components. Because of difficulty in estimating parameters the alternative was left pending time of more abundant data. The specific probability parameter for extreme load part of outer shell components was chosen as the only new model parameter because of its generally higher importance.

A2-3 Minimal Cut Sets and Probability Reduction

This section explains handling of failure combinations for localized CCFs of control rods, exemplified by a practical case.

A2-3.1 Minimal Cut Set presentation

Generic acronym MCS is used here for Minimal Cut Set. It is linked to different placements of Minimal Critical Shape, representing corresponding failure event combinations which are denoted by using rod positions.

Minimal Critical Shape of 3x2 rods and band correlation type are used here as an example owing to possibility of easy manual reduction of MCS presentation. Figure A2-3 shows one MCS, i.e. one possible placement of Minimal Critical Shape, compare to shell layout description in Report Section 6.4 and Figure 6-2. There are three possible placements of Minimal Critical Shape (confined in inner and outer shells) differing by linear translation. Correspondingly, MCSs are following:

$$\begin{aligned} M_1 &= X_0X_1X_2X_3X_4X_5 \\ M_2 &= X_0X_1X_2X_5Y_1Y_2 \\ M_3 &= X_0X_3X_4X_5Y_3Y_4 \end{aligned} \qquad \text{(A2-11)}$$

It is convenient to present the total failure probability in terms of Psg entities by using so called combination matrix, because then the condition of other rods beyond each combination and especially beyond the defined outer shell need not be considered. This unburdens the quantification substantially. It should be emphasized, that Psg entities are defined as the probability of specific components failing irrespective of the condition of other components (they may either operate or fail).

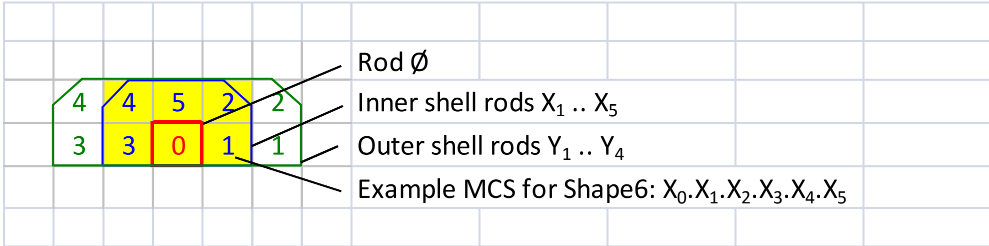

Figure A2-3 Example with Minimal Critical Shape of 3x2 rods and band correlated CCFs.

A2-3.2 Derivation of combination matrix

The derivation of combination matrix starts from the probability expression based on MCS presentation:

$$P_{TOP} = P(M_1 - M_2 + .. + M_N) \qquad \text{(A2-12)}$$

where M_i denotes an individual MCS. The exact reduction is given by the following formula of alternate positive and negative terms. i.e. by so called Inclusion-Exclusion Principle [Henley&Kumamoto, p.320]:

$$P_{TOP} = S_1 - S_2 + .. + (-1)^{N-1} . S_N \qquad \text{(A2-13)}$$

where

$$S_1 = \sum_{1 \le k \le N} P\{M_k\}$$

$$S_2 = \sum_{1 \le k_1 < k_2 \le N} P\{M_{k_1} . M_{k_2}\}$$

..

$$S_m = \sum_{1 \le k_1 <..< k_m \le N} P\{M_{k_1} ... M_{k_m}\}$$

..

$$S_L = \Pr\{M_1 . M_2 .. M_N\}$$

The reduced presentation is a list of probability terms with positive or negative sign. Each term constitutes of the probability of a set of basic events with a specific number of inner shell components k_{is} and outer shell components k_{os} plus failure of Rod ∅ that is contained in every MCS. When the probability terms are sorted and summed according to multiplicity indexes, the total failure probability can be expressed in the following way:

$$P_{TOP} = \sum_{k_{is}=0}^{n_{is}} \sum_{k_{os}}^{n_{os}} Cmb(k_{is}, k_{os}) \cdot Psg(k_{is}, k_{os}) \qquad (A2\text{-}14)$$

where

$Cmb(k_{is}, k_{os})$ = *Combination matrix*

$Psg(k_{is}, k_{os})$ = *Probability of specific k_{is} inner shell components and specific outer shell components k_{os} and Rod ∅ failing*

It is worthwhile to notice that no truncations or other approximations are involved in this stage of quantification. The precision of the results is limited only by the numerical accuracy of practical calculation.

A2-3.3 Example reduction

The reduction of MCS presentation is shown in Table A2-1 for the example. Only three levels S_1, S_2 and S_3 are relevant. The derived combination matrix is rather simple: it is presented on the calculation sheet, Figure A2-4. Elements (5,4) cancel each other.

Table A2-1 Reduction of MCS presentation for Minimal Critical Shape of 3x2 rods, band correlated case.

	Minimal Cut Set			Basic events										Psg element		
				Inner shell						Outer shell						
Level	M1	M2	M3	0	1	2	3	4	5	1	2	3	4	Kis	Kos	Sign
S1	1			1	1	1	1	1	1					5	0	+
		1		1	1	1			1	1	1			3	2	+
			1	1			1	1	1			1	1	3	2	+
S2	1	1		1	1	1	1	1	1	1	1			5	2	-
	1		1	1	1	1	1	1	1			1	1	5	2	-
		1	1	1	1	1	1	1	1	1	1	1	1	5	4	-
S3	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4	+

It should be noticed that the number of MCSs is generally distributed along diagonal of combination matrix with $k_{is} + k_{os} = k_{cri}$, size of Minimal Critical Shape. The overall sum of the elements shall equal to one, which also provides an important check.

In cases as simple as the example, the reduction can well be managed manually. In more complex cases a computerized reduction algorithm can be used. The reduction work rapidly escalates as the function MCS complexity. For example, in Olkiluoto 1 and 2 case, five adjacent rods being critical and radial correlation type with of 8 inner shell and 12 outer shell components, the reduction of probability function of 40 MCSs took 15 minutes by a PC with i486/66 in the beginning of the 90'ies. The corresponding execution time with nowadays' desktop computer is less than one second, i.e. speed increase of three orders of magnitude.

The example case is quantified by using the data of reference application to Forsmark 1 and 2 [SKI Report 2006:05]. The calculations are presented in Figure A2-4. It is of interest to show the quantitative contributions according to $S_\#$ level:

Table A2-2 $S_\#$ level contributions in the example case.

Level	Term sum	Cumulative	Relative
S_1	4.95E-4	4.95E-4	102.7%
S_2	-1.53E-5	4.80E-4	99.5%
S_3	2.42E-6	4.82E-4	100.0%
TOP	4.82E-4		

The result is dominated by first MCS because it is totally contained in inner shell while the other two MCSs contain two outer shell components. Compare to Psg(5, 0) = 4.82E-4 (coincidentally very close to P_{TOP}) in the results array shown in Figure A2-4. Usually in practical cases either the first order approximation S_1 (generally valid upper limit) would be overly conservative and second order approximation S_1 - S_2 (generally valid lower limit) is overly optimistic. Exact probability derivation is thus crucial.

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F1/F2 - CRDAs, screw insertion exclusively, band correlated CCFs, Shape 6 example

CCF group size		CLM parameters				09.11.2015 17:53
KisMax	5	p_tot	9.02E-2	c_co	0.28	
KosMax	4	p_xti	2.0E-4	c_cx	0.70	
Kcri	6	p_xto	2.0E-5	*u_out*	*3.46*	

Cmb

Inner shell	Outer shell components Kos= 0	1	2	3	4
Kis=0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	2	0	0
4	0	0	0	0	0
5	1	0	-2	0	0

Element sum: 1

Psg

Inner shell	Outer shell components Kos= 0	1	2	3	4
Not X0	1.00E+0	3.27E-4	8.84E-6	3.80E-6	2.48E-6
Kis=0	9.01E-2	1.33E-4	7.96E-6	3.76E-6	2.47E-6
1	1.75E-2	7.00E-5	7.43E-6	3.73E-6	2.46E-6
2	5.17E-3	4.41E-5	7.07E-6	3.69E-6	2.45E-6
3	1.98E-3	3.17E-5	6.82E-6	3.66E-6	2.44E-6
4	9.11E-4	2.49E-5	6.62E-6	3.64E-6	2.43E-6
5	4.82E-4	2.09E-5	6.46E-6	3.61E-6	2.42E-6

P_{TOP} 4.82E-4

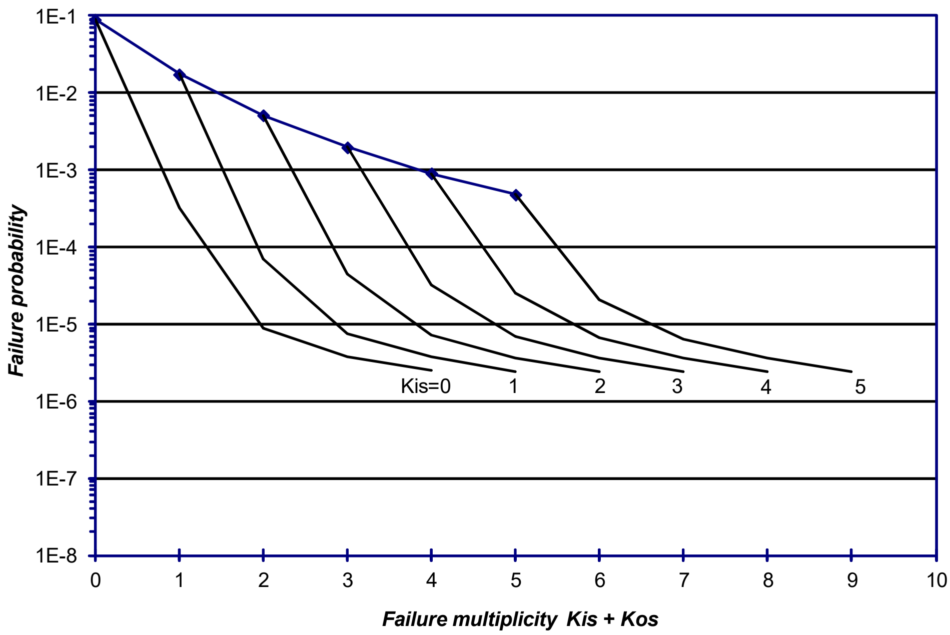

Figure A2-4 Quantification results in the example case, Minimal Critical Shape of 3x2 rods, band correlated CCFs.

A2-3.4 Presentation of quantification results

The quantification results as presented for the example case in Figure A2-4 show the calculated Psg array, and functional failure probability P_{TOP} = 4.82E-4. Row 'Not X0' shows $Psg_{LocZ}(-1, k_{os})$ values, representing virtual cases where no inner shell component fails neither Rod ∅, compare to the discussion in Section A2-2.1.

The thick curve in the diagram shows the failure probability of inner shell components, and thin lines the mixed cases where a specific number of inner shell components fail together with one or more outer shell components.

The result of the example is not sensitive with respect to new parameter p_xto, because dominated by first MCS as already discussed. In many practical cases the results use to be crudely linearly dependent of p_xto.

A2-4 References

CLM_LocZ_1995
Mankamo, T., Application of Common Load Model to localized CCF mechanisms of control rods. Work notes, 20 December 1995.

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Common Cause Failure Analysis of Control Rods and Drives in the Swedish and Finnish BWR plants, Operating Experiences in 1983-2003. Prepared by T. Mankamo, Avaplan Oy, NPSAG 05-003 Project. SKI Report 2006:05, November 2006.

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