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Research

Scattering of SH Waves by Isolated Cracks Using a Hybrid Approach

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SKI perspective

Background

Mathematical modelling is an important tool for developing NDT systems and in the end to get more reliable testing situations. It is also important in the situation of inspection qualification to get more flexibility and cost effectiveness.

SKI has for the last decade been supporting research for development of a model for ultrasonic testing. SKI sees the importance and the benefits in modelling testing situations. This project is a fist step to be able to simulate a situation with a more complicated geometry of the modelled defect.

Purpose of the project

The purpose of the project is to develop a two dimensional hybrid model for simulation of ultrasonic scattering from a crack like structure. The result of the project will give us more information about how to get on with the 3D case and implementation in the UTDefect software.

Results

The results show that the two-dimensional model studied, can be generalized into three-dimensions. The results also show where special attention has to be given when expanding the model into three dimensions.

Project information

Responsible for the project at SKI has been Peter Merck.
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Scattering of SH Waves by Isolated Cracks Using a Hybrid Approach

A. Jonas Niklasson
Gert Persson

Department of Material and Manufacturing Technology
Chalmers University of Technology
SE-412 96 Göteborg
Sweden

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Summary

A two-dimensional hybrid method, combining finite element (FE) and an boundary element method (BEM), is used to study ultrasonic waves.

The main objective of this preliminary study is to develop a mathematical model of the ultrasonic scattering from an isolated general-shaped crack. In order to achieve such a model a hybrid method is developed, the method couple FE and the somewhat less numerical demanding integral equations.
This project is also an investigation whether it is possible to develop such a two-dimensional hybrid model of the complete NDT situation. If this preliminary study is successful the continuation would be a three-dimensional model that could be incorporated into the UTDefect© software.

The hybrid model, together with a two-dimensional SH-probe, is used to study scattering from three cases: the case of no defect, one circular and one star-shaped defect.
The results show that is possible to make a two-dimensional model in such a way that it can be generalized into three-dimensions. The investigation also shows that the well known numerical dispersion, of finite-element computations at high wavenumbers, has to be given special attention.

Sammanfattning

Rapporten presenterar en tvådimensionell hybridmetod som kombinerar finita element (FE) med en randelementsmetod (BEM) för elastiska vågutbredningsproblem. Metoden används för att bygga en modell där en isolerad defekt ligger i ett område med finita element, detta område kopplas till omgivningen via randintegralselement.

Studien är främst en förstudie för att undersöka om det är möjligt att modellera sprickor med komplicerad geometri mha FE och därefter koppla detta område till en i viss mening mindre beräkningskrävande integralrepresentation.
Studien syftar även till att undersöka om det är möjligt att modellera en realistisk OFP-situation med den föreslagna hybridmodellen. På sikt skulle en sådan defektmodell kunna implementeras i den befintliga programvaran UTDefect©.

Med den framtagna hybridmodellen studeras spridning av ultraljud från en tvådimensionell SH-sökare för tre fall: fallet med ingen defekt, en cirkulär och en stjärnformad defekt. De numeriska resultaten visar att det är möjligt att bygga en fungerande tvådimensionell modell på ett sådant sätt att den kan generaliseras till tre dimensioner. Undersökningen visar också att problemen med numerisk dispersion, för finita elementlösningen, måste ägnas speciell uppmärksamhet.

Introduction

In industry non destructive testing (NDT) is a commonly used method to evaluate the integrity of individual components. In-service induced cracks such as fatigue and stress corrosion cracks can, if they are detected, be sized and monitored in order to postpone repairs or replacements. The reliability of a NDT method is highly dependent on how the equipment is adjusted to a specific object and to anticipated crack features. The crack feature and morphology vary widely between different crack mechanisms and between material types, in which the crack appear.

An infinite number of variables and possibilities have to be reduced into a limited group of statistically relevant NDT situations. The qualification of inspection systems includes the reliability to detect, locate, characterise and accurately determine the size of defects that may occur in the specific type of component. Despite the fact that the proposed qualification procedure with test pieces is very expensive it also tends to introduce a number of possible misalignments between the actual NDT situation that is to be performed and the proposed experimental simulation. Apart from the problem of reconstructing the geometry and material, the fabricated defects also has to be introduced with a verified prescription of its size and NDT characteristics.

In an ENIQ document (European Network for Inspection Qualification, [1]) the definition of whether a flaw can be regarded as realistic only states the necessity of a resemblance in the signal response from a real flaw. This indicates different demands of the manufactured specimens as the NDT system is to be used in connection with detection, sizing or characterisation. This also means that the quality of the test piece is bound to be associated with a specific NDT method.

The manufacturing of specimens with induced artificial flaws introduce a number of parameters (location, size and orientation) that are difficult to control and verify without employing destructive testing. Implants of service-induced real flaws introduce other parameters that are not trivial to control, e.g. the effect on surrounding material.

According to the Swedish Nuclear Power Inspectorate's requirements in the regulations concerning structural components in nuclear installations, in-service inspection must be performed using inspection methods that have been qualified. These demands on reliability of used NDE/NDT procedures and methods have stimulated the development of simulation tools of NDT. To qualify the procedures extensive experimental work on test blocks is normally required. A thoroughly validated model has the ability to be an alternative and a complement to the experimental work in order to reduce the extensive cost that is associated with the previous procedures.

Up till now only a couple of models have been developed that cover the whole testing procedure, i.e. they include the modelling of transmitting and receiving probes, the scattering by defects and the calibration. Chapman [3] employs geometrical theory of diffraction for some simple crack shapes and Schmitz et al [4] develops a type of finite integration technique for a two-dimensional treatment of various defect types. These models are compared with experiments within the PISC project by Lakestani [5]. Overviews of the modelling of ultrasonic NDT are given by Gray et al [6] and Achenbach [7].

The UTDefect software has been developed at the Dept. of Mechanics at Chalmers University of Technology ([8], [9],…,[14]) and has been experimentally validated and verified [12]. The software simulates the whole testing procedure with the contact probes (of arbitrary type, angle and size) acting in pulse-echo or tandem inspection situations. There is a broad variety of simple-shaped defect types included in the program and roughness and different spring boundary conditions on the crack surfaces can be added to some of the defects. This model employs various integral transforms and integral equation techniques to model probes and the scattering by defects. In this way the frequency and some geometry limitations of the geometrical theory of diffraction (GTD) [3], [15] are avoided, still without the computer requirements that would result from a volume discretization using the finite element method (FEM) or the finite integration technique [4].

Using semi analytical methods, as the integral equation technique above, limits the possibilities to model the geometry of the defect. In most cases the simple-shaped defects like: spheres, strip-like or penny-shaped cracks is believed to model the real test situation good enough. However, sometimes the actual shape of the defect is of great importance. The stress corrosion cracks (SCC) often tends to have a heavily branched macroscopic shape with a large number of crack tips. The diffraction from the crack tips is commonly used as the basis for the defect size analysis and as a consequence, ultrasonic NDT methods are not always reliable in this kind of applications. Cracks of branched Y-shape have successfully been investigated in two-dimensions with methods as GTD [16] and hybrid methods [17], here for surface breaking cracks.
Numerical approaches, although costly, can be used to evaluate much more complicated crack geometries than the Y-shape. In a realistic NDT situation the distance of transportation of energy from defect to probe are large and relatively easy to define. Using a numerical method to compute this transportation, with almost no change in information, is unnecessarily costly. Semi-analytical methods, on the other hand, are ideal to handle far-field problems.

The above mentioned hybrid methods take advantage of both semi-analytical and numerical approaches. The basic idea is to surround the defect by a finite element scheme and deal with the propagation between the probe and the defect with a semi analytical method. In this way it is possible to model more complex crack geometries that involves a complicated scattering processes without getting to large numerical models. Where it is possible to implement, semi-analytical and fully numerical approaches are complementary.

The hybrid method used in [17] [18] [19] combines the finite element discretization of the near field with a boundary integral representation of the far field for two-dimensional problems. Here the interior region with finite elements slightly overlaps the exterior region and couples using a boundary integral representation. The coupled procedure of the boundary element method (BEM) and the finite element method (FEM) are frequently studied to evaluate singular or hypersingular integrals. The techniques used for two-dimensional problems in [20] [21] [22] extend, more or less directly, to three-dimensional problems. Inspired by [20] we adopt this approach to couple FEM and BEM.

Hybrid models have also been implemented in simulation platforms like CIVA [23], here the far field is represented by a ray-method and the defect lays within a parallelepipedic box of finite elements.

The main objective of this project is to develop a mathematical model of the ultrasonic scattering from an isolated general-shaped crack. In order to achieve such a model we have chosen to work with a hybrid method that couple FEM and BEM.
Investigating the less complicated two-dimensional SH-case with a technique that extend to three-dimensions makes it possible to estimate the amount of work to solve the latter. Here SH-waves are particularly attractive because of the simple nature of their interaction with defects. The response of SH-waves can be clearly observed and the mechanism of the scattering can be easily understood. This is important for the future investigation of more complicated scattering problem.
If this preliminary study is successful the continuation would be a three-dimensional model that could be incorporated into the UTDefect© software. A mathematical model would enable parametric studies of the influence the actual shape of a crack has on ultrasonic detectability and its effectiveness in sizing of these kinds of defects.

The proposed hybrid method is not only of interest for scatterers of complicated geometry or branched cracks. It should be possible to deal with problems of multiple scattering by a cluster of scatterers or by scatterers inbedded in a different material from the surrounding matrix in the same way as the problem of a single scatterer in a homogeneous material. Possible developments can also include surface breaking defects. The user of a simulation platform, based on this hybrid method, would be able to describe a quite arbitrary defect geometry and corresponding elastic properties.

Hybrid approach to ultrasonic scattering

The scattering of two-dimensional anti-plane (SH) waves by a defect in an isotropic component is considered in this report. The scattering problem at hand is depicted in Fig. 1. A known wave $u^{(\mathrm{i})}$ is incident upon a scatterer S and the aim is to compute the total field $u = u^{(\mathrm{i})} + u^{(\mathrm{s})}$, here $u^{(\mathrm{s})}$ is the scattered field.

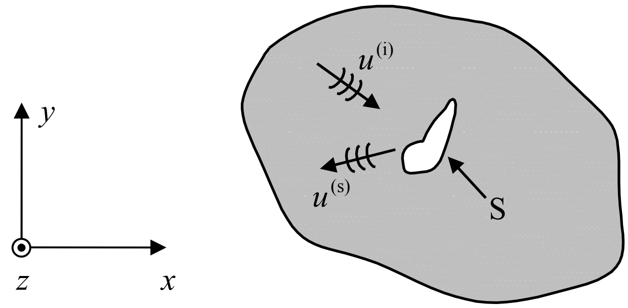

Figure 1: The scattering problem.

Time-harmonic conditions are assumed and since the problem only has one displacement component, the field may be written as $u(x, y)\mathrm{e}^{-\mathrm{i}\omega t}$. The time-harmonic equation of motion is then given by

$$\nabla^2 u + k_{\mathrm{T}}^2 u = 0 \,. \tag{1}$$

The time factor $\mathrm{e}^{-\mathrm{i}\omega t}$ is suppressed throughout, $k_{\mathrm{T}} = \omega / c_{\mathrm{T}}$ is the transverse wave number and $c_{\mathrm{T}} = \sqrt{\mu / \rho}$ is the transverse wave speed.

In order to enable a rather general shape of the scatterer S, a hybrid approach is used to solve the scattering problem. The approach is based on a description of the problem by means of a partial differential equation inside the domain embedding the scatterer and an integral representation of the field in the exterior domain. The method is a hybrid between the finite element method (FEM) and the boundary element method (BEM). The main motivation for this choice of methods is that FEM can handle general geometric object efficiently and BEM is efficient for large domains. Below, FEM and BEM are described for the problem and finally, the approach used to combine them.

The finite element method - FEM

In a domain Ω_1 enclosing the scatterer (see Fig. 2), the finite element method (FEM) is used to discretize the equations of motion.

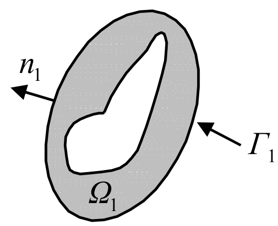

Figure 2: The FEM domain Ω_1.

First, the equation of motion is multiplied by a test function $v(x,y)$ and then integrated over the domain Ω_1. The result is

$$\int_{\Omega_1} (\nabla u \cdot \nabla v - k_{\mathrm{T}}^2 \mathrm{uv})\,\mathrm{d}\Omega = \int_{\Gamma_1} \frac{\partial u}{\partial n_1} v\,\mathrm{d}\Gamma\,, \tag{2}$$

where Γ_1 is the boundary of Ω_1 and n_1 the outward directed normal to Γ_1. Next, triangulations of the domain Ω_1 and its boundary Γ_1 are introduced. The displacement field u and its normal derivative $\partial u / \partial n_1$ are then discretized as

$$u(x,y) = \sum_{m=1}^{N_\Omega} U_m\,\varphi_m(x,y), \qquad \frac{\partial u(x,y)}{\partial n_1} = \sum_{m=1}^{N_\Gamma} T_m\,\psi_m(x,y)\ . \tag{3}$$

Above, $\psi_m(x,y)$ is the restriction of the shape functions $\varphi_m(x,y)$ to the boundary Γ_1. N_Ω is the number of nodes in the triangulation of Ω_1, and N_Γ is the number of nodes in the triangulation of Γ_1. Letting the test function $v(x,y) = \varphi_j(x,y), j = 1,\ldots,N_\Omega$, a linear system of equations is obtained as

$$\left.\begin{aligned}
&(K_\Omega - k_T^2 M_\Omega)U = M_\Gamma T \\
&(K_\Omega)_{jm} = \int_{\Omega_1} \nabla\varphi_j \cdot \nabla\varphi_m\,\mathrm{d}\Omega \\
&(M_\Omega)_{jm} = \int_{\Omega_1} \varphi_j\,\varphi_m\,\mathrm{d}\Omega \\
&(M_\Gamma)_{jm} = \int_{\Gamma_1} \psi_j\,\psi_m\,\mathrm{d}\Gamma
\end{aligned}\right\} \tag{4}$$

A homogeneous, isotropic continuum is nondispersive. This is not the case for the discrete finite element method, the solution is anisotropic in the sense that it depends on the orientation of the mesh with respect to the direction of propagation. This is a well known phenomenon that can be somewhat treated by the Galerkin/Least-Squares Method [27].

The boundary element method - BEM

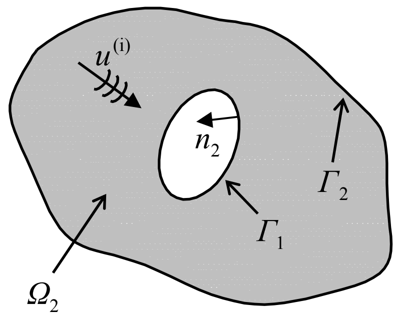

Figure 3: The BEM domain.

The displacement field in the domain exterior to Ω_1, Ω_2, is evaluated using a boundary integral representation

$$\int_{\Gamma_1} \left(u(\boldsymbol{r}) \frac{\partial G}{\partial n_2}(\boldsymbol{r};\boldsymbol{r}') - \frac{\partial u}{\partial n_2}(\boldsymbol{r})\, G(\boldsymbol{r};\boldsymbol{r}') \right) \mathrm{d}\Gamma + u^{(i)}(\boldsymbol{r}') = \begin{cases} u(\boldsymbol{r}'), & \boldsymbol{r}' \in \Omega_2, \\ 0, & \boldsymbol{r}' \notin \Omega_2. \end{cases} \tag{5}$$

Here n_2 denotes the outward directed normal to the boundary Γ_1 of the domain Ω_2 and $G(\boldsymbol{r};\boldsymbol{r}')$ is the Green's function in the domain enclosed by Γ_2. If the domain is infinite, $G(\boldsymbol{r};\boldsymbol{r}')$ is the free space Green's function given by

$$G(\boldsymbol{r};\boldsymbol{r}') = \frac{\mathrm{i}}{4} \mathrm{H}_0^{(1)}(k_\mathrm{T} |\boldsymbol{r} - \boldsymbol{r}'|), \tag{6}$$

where $\mathrm{H}_0^{(1)}$ denotes the Hankel function of the first kind and order zero.

If the load point $\boldsymbol{r}'$ approaches the boundary Γ_1 from Ω_2, only boundary data are present in the boundary integral representation Eq. (5). This equation is called a boundary integral equation. Calculation of the integrals by boundary discretization leads to algebraic equations for solving unknown boundary data in terms of known boundary data. The displacement u and its normal derivative $\partial u / \partial n_2$ are represented as

$$u(x,y) = \sum_{m=1}^{N_\Gamma} U_m \psi_m(x,y), \qquad \frac{\partial u(x,y)}{\partial n_2} = \sum_{m=1}^{N_\Gamma} T_m \psi_m(x,y). \tag{7}$$

The representations are inserted into the integral equation, the result is multiplied by $\psi_j(x,y)$, $j = 1,\ldots,N_\Gamma$, and integrated over Γ_1. The resulting linear system of equations is (note that the relation $n_2 = -n_1$ has been used)

$$
\begin{aligned}
&J_\Gamma U + I_\Gamma T + F^{(\mathrm{i})} = M_\Gamma U, \\
&(J_\Gamma)_{jm} = \lim_{\Gamma_1' \to \Gamma_1} \int_{\Gamma_1'}\int_{\Gamma_1} \psi_m(\mathbf{r}) \frac{\partial G}{\partial n_2}(\mathbf{r};\mathbf{r}')\psi_j(\mathbf{r}')\,\mathrm{d}\Gamma \mathrm{d}\Gamma', \\
&(I_\Gamma)_{jm} = \lim_{\Gamma_1' \to \Gamma_1} \int_{\Gamma_1'}\int_{\Gamma_1} \psi_m(\mathbf{r})\, G(\mathbf{r};\mathbf{r}')\psi_j(\mathbf{r}')\,\mathrm{d}\Gamma \mathrm{d}\Gamma', \\
&(F^{(\mathrm{i})})_j = \int_{\Gamma_1} u^{(\mathrm{i})}(\mathbf{r}')\psi_j(\mathbf{r}')\,\mathrm{d}\Gamma'.
\end{aligned}
\tag{8}
$$

For evaluating the boundary integrals J_Γ and I_Γ appearing in Eq. (8), the approach in [20] is used. Here the singularities of the Green's function require careful analysis when the load point $\boldsymbol{r}'$ approaches the boundary.

The hybrid method

Based on the discretization of the displacement field and its normal derivative by means of FEM and BEM, a hybrid scheme may be derived to solve the scattering problem. First, the coefficients of the normal derivative of *u* may be obtained from Eq. (8) as

$$T = I_\Gamma^{-1}(M_\Gamma - J_\Gamma)U - I_\Gamma^{-1}F^{(\mathrm{i})} . \tag{9}$$

This expression is then inserted into the FEM system of equations Eq. (4) and the final result is

$$(K_\Omega - k_T^2 M_\Omega - M_\Gamma I_\Gamma^{-1}(M_\Gamma - J_\Gamma))U = -M_\Gamma I_\Gamma^{-1} F^{(\mathrm{i})} . \tag{10}$$

Once the resulting system of equations has been solved, the solution in the FEM domain is known. By computing *T* from Eq. (9), the displacement field in the BEM domain may be computed from the boundary integral representation Eq. (5).

The incident field

The incident field is taken as the field from an ultrasonic SH-transducer. To model the contact probe in this isotropic case the approach by Boström and Wirdelius [24] is employed. This approach have been used in the 2D case by Niklasson [25], but then generalized to the more complicated anisotropic case.

The probe is situated on an elastic half-space that has a traction-free surface except beneath the probe where the traction is assumed to be known. The coordinate system is introduced according to Fig. 4.

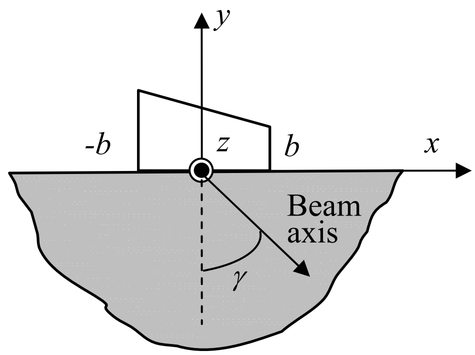

Figure 4: The geometry of the probe.

The probes index point is at the origin and the beam axis is in the fourth quadrant in the xy-plane, making an angle γ with the negative y axis. The probe is in contact with the half-space at $x \in [-b, b]$ and $y = 0$. The traction beneath the probe is taken as a slight modification of the traction from a plane wave propagating in the direction of γ :

$$(\sigma_{yz})_{y=0} = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = -A\, g(x)\, i \mu k_{\mathrm{T}} \cos(\gamma)\, e^{-\mathrm{i} k_{\mathrm{T}} x \sin\gamma}. \tag{11}$$

In Eq. (11), the function $g(x)$ is zero outside the probe and if a piston model is used, one beneath it, i.e.,

$$g(x) = \begin{cases} 1 & x \in [-b, b], \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The displacement field in the half-space due to the probe may be calculated by means of Fourier techniques. If the piston model is used, the field is given by

$$\begin{aligned} u^{(\mathrm{i})}(x, y) &= \int_{-\infty}^{\infty} \frac{A k_{\mathrm{T}} \cos\gamma \, \sin((q - k_{\mathrm{T}} \sin\gamma) b)}{\pi (q - k_{\mathrm{T}} \sin\gamma)\, h(q)} e^{\mathrm{i}(qx - h(q) y)}\, \mathrm{d}q, \\ h(q) &= \sqrt{k_{\mathrm{T}}^2 - q^2}, \quad \operatorname{Im} h(q) \geq 0. \end{aligned} \tag{13}$$

If the defect is far away (many wavelengths away) from the probe, the expression for the displacement field Eq. (13) may be approximated by the method of stationary phase. The approximate incident displacement field is then given by

$$\begin{aligned} u^{(\mathrm{i})}(x, y) &= \frac{A b \cos\gamma \sin(k_T\, (x / r(x, y) - \sin\gamma) b)}{k_{\mathrm{T}}\, (x / r(x, y) - \sin\gamma)} \sqrt{\frac{2 k_{\mathrm{T}}}{\pi r(x, y)}}\, e^{i (k_{\mathrm{T}} r(x, y) - \pi / 4)}, \\ r(x, y) &= \sqrt{x^2 + y^2}. \end{aligned} \tag{14}$$

Numerical results

In this section, a few numerical examples are presented.

In most cases, with automatic mesh generation and refinement in mind, triangular elements are preferred for two-dimensional geometries. The elements used in this study are quadratic triangles and the mesh is unstructured as shown in figure 6a, 7a and 8a. The diameter of the FE-area is three times the centre wavelength λ_c, which corresponds to the centre frequency used in both the probe-model and for the plane wave.

Figure 6a shows the FE-mesh for the case of no defect which is used as a reference and for making sure that the waves do not get distorted by the discretization. The case of a circular defect is shown in figure 7a, where the diameter of the defect is 1.2 times the centre wavelength. The star-shaped defect of figure 8a has an outer diameter of 1.8 times the centre wavelength and an inner diameter of 0.6. For the presented cases there are at least 10 elements per centre wavelength.
The three geometries are studied for two different incident waves, one from the SH-probe and one for an incident plane wave. The displacement field is plotted for two different times, the first when the waves just have passed the defect and the second when the incident field is barely not noticeable, but movements of the defect still give a scattered field.

The grayscale –20 to 0 dB is normalized according to the maximum value for the case of no defect, which then is 0 dB. The normalization are made for the incident plane wave and the probe wave, respectively.

In figure 6b, 6c, 7b, 7c, 8b and 8c the incident wave is given by the SH-probe model, the angle of the probe is $\gamma = -30^{\circ}$ and the location of the probe is such that the beam axis hits the center of the FE-area. The probe have the size $b = 3\lambda_c$, se figure 4. The center of the FE-area is at a depth of 60 centre wavelength λ_c and the probe model used is given by the farfield expression. Figures (b) show the first time and figures (c) the later.

In figure 6d, 6e, 7d, 7e, 8d and 8e the incident wave is given by a plane wave, the angle of incidence is -30°. Figures (d) show the first time and figures (e) the later.

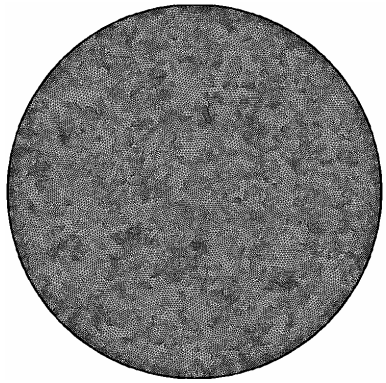

Figure 6a: The FE-mesh in an area without defect.

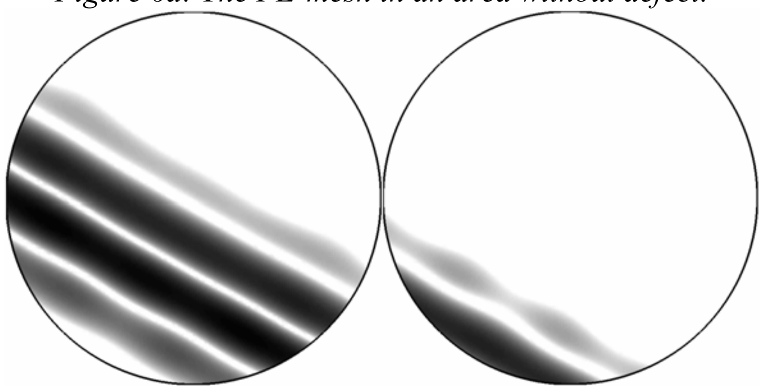

Figure 6b, c: The displacement field in the FE-area without defect, SH-probe model.

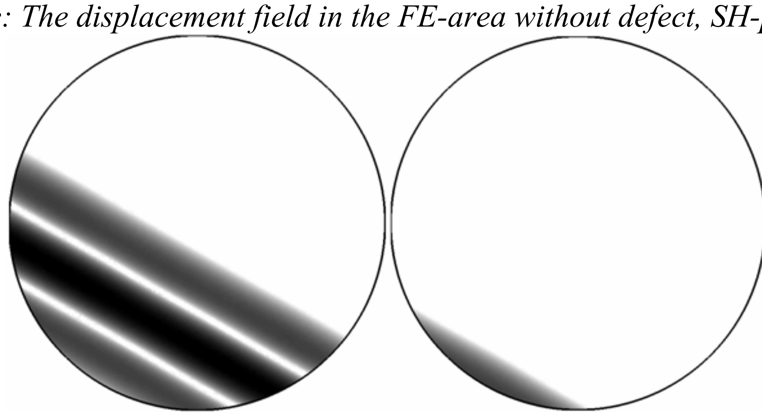

Figure 6d, e: The displacement field in the FE-area without defect, incident plane wave.

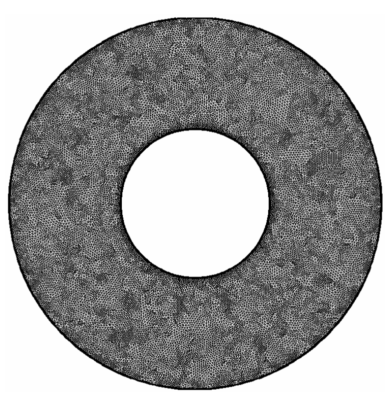

Figure 7a: The FE-mesh in an area containing a circular crack.

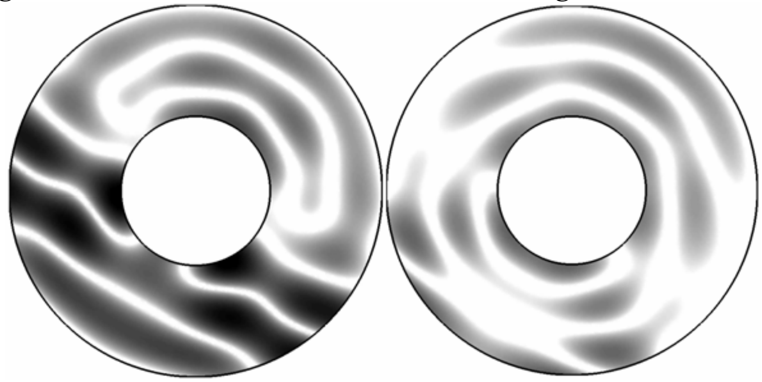

Figure 7b, c: The displacement field in the FE-area containing a circular crack, SH-probe model.

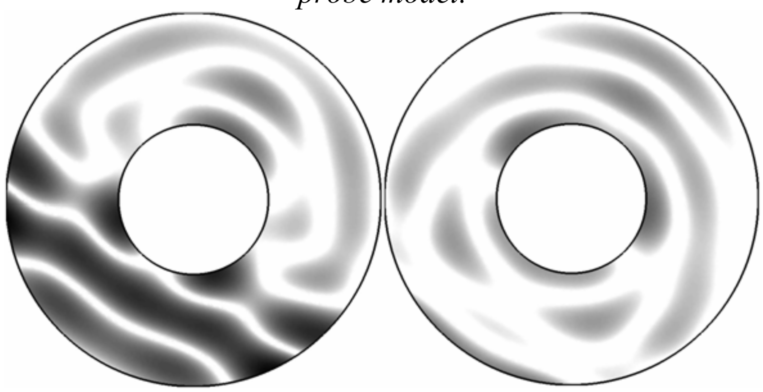

Figure 7d, e: The displacement field in the FE-area containing a circular crack, incident plane wave.

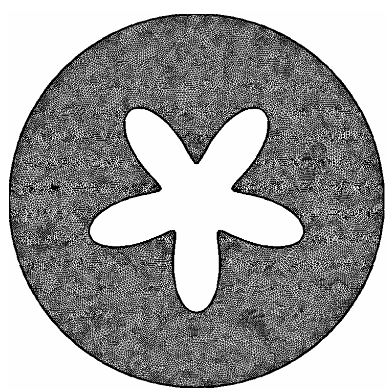

Figure 8a: The FE-mesh in an area containing a star defect.

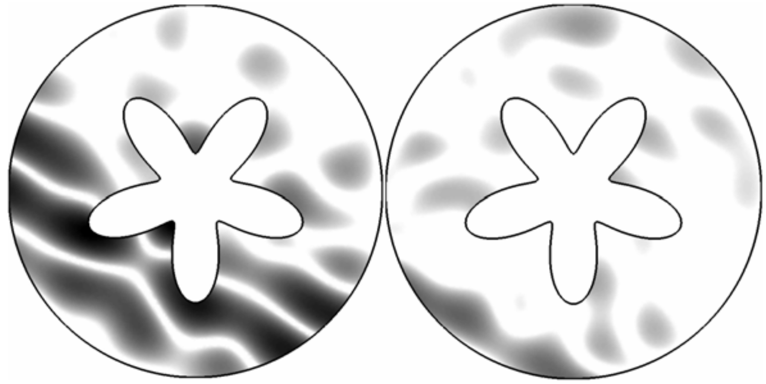

Figure 8b, c: The displacement field in the FE-area containing a star defect, SH-probe model.

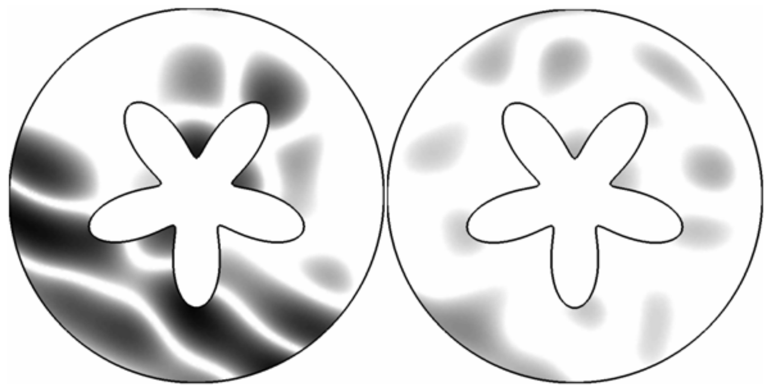

Figure 8d, e: The displacement field in the FE-area containing a star defect, incident plane wave.

Since neither crack opening displacements or stress intensity factors are calculated, there are no need for a thorough investigation of crack tip behavior. With a large number of quadratic type element near the tip the correct square root singularity will be obtained. The two main reasons for using quadratic triangles is that, the number of finite elements have a small effect on the total computation time, and for the reason of flexibility. All kinds of geometrically complicated scatterer are treated in the same way.

Fig. 9 shows the displacement field at fixed frequency in a half-space containing a circular crack (the innermost circle). The boundary Γ_1 separating the two solution techniques (FEM and BEM) is circular (the outermost circle) and shown in the figure. In the figure 9a, the probe is unangled ($\gamma = 0^\circ$) with the probe located straight over the crack. In figure 9b and 9c, the angle of the probe is $\gamma = 14^\circ$ and the location of the probe is such that the beam axis hits the crack. The center of the probe is then in the upper left corner of figure 9b and 9c. All figures shows the expected symmetry about the central beam axis

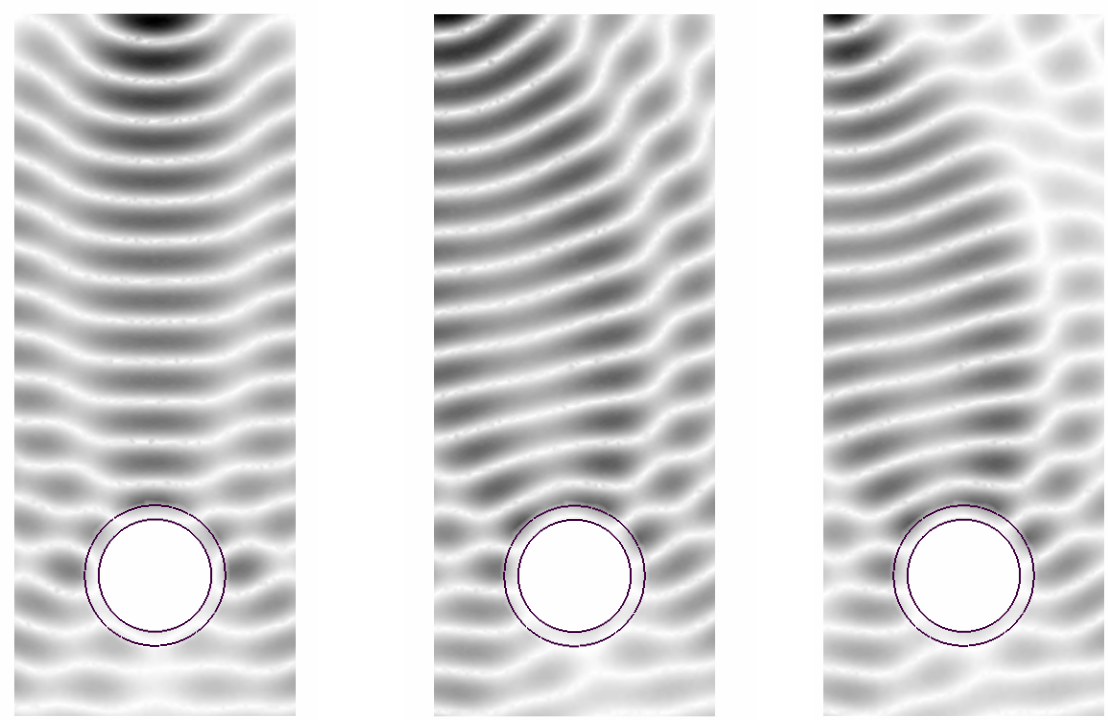

Figure 9a, b, c: The displacement field in a component containing a circular crack.

It should be noted that the numerical computation of the field in the BEM domain and the computation of the field on the boundary Γ_1 takes approximately half of the total computation time each, for the calculations in figure 9 it takes even more than half of the total computation time. For standard calculations, where the signal response $\delta\Gamma$ [26] is calculated, there is no need to calculate the field in the entire BEM region. The FEM calculations, in the interior domain, have a very small effect on the total computation time.

Concluding remarks

The scattering of SH-waves by an isolated defect has been investigated numerically by a hybrid method which combines the finite element method (FEM), the boundary element method (BEM) and a boundary integral representation. The incident field is taken as the field from an ultrasonic SH-transducer and due to the simple nature of the SH-wave, each individual diffracted wave can in principle be identified and observed.

The proposed hybrid method shows to be both efficient and suitable for solving ultrasonic scattering from defects of complicated geometry. The FEM is a good choice for describing detailed geometry and is effective in numerical computations in a narrow region. As expected, calculation of entities on the boundary region in the BEM domain takes a lot of computation time. The discretization of the boundary region have to be made very careful, the FE- discretization of the interior region can be made more rough since its impact on the total computation time is small. It is clear that the hybrid method is better than solitary using the BEM. The latter would produce a model where the number of elements, and thereby computation time, increases not only with frequency but also with the complexity of the geometry. Computation time could then be determined by geometry and not frequency.

Combining the FEM and the BEM is, due to numerical dispersion, not without difficulties. The FE-computed solution will gradually get out of phase with the integral equation solution and cause difficulties on the coupled boundaries. In practical terms, this leads to an increase in the cost of the FE solution at higher wavenumbers. For the studied problem the Galerkin/least-squares method has been successfully employed for linear triangles.

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STATENS KÄRNKRAFTINSPEKTION
Swedish Nuclear Power Inspectorate

POST/POSTAL ADDRESS SE-106 58 Stockholm
BESÖK/OFFICE Klarabergsviadukten 90
TELEFON/TELEPHONE +46 (0)8 698 84 00
TELEFAX +46 (0)8 661 90 86
E-POST/E-MAIL ski@ski.se
WEBBPLATS/WEB SITE www.ski.se